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Upper limit of height-spacing ratio for transverse aeolian bedforms

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Abstract – An effective “minimum force” hypothesis is proposed for the evolution of aeolian bedforms. The upper limit of the height-spacing ratio suitable for ripples, small-scale dunes and giant dunes is obtained from fluid dynamics by using this hypothesis.

The aeolian bedforms may range in height from 0.05 cm to 500 m and in spacing or wavelength from 0.5 cm to 5500 m [1–5]. The mechanisms responsible for the formation, evolution and dynamics of aeolian sand accumulations at different scales are widely thought to be different. For example, a) the sand ripple wavelength is determined by the mean length of saltation or reptation [6,7]; b) the initial size of small-scale dunes depends on the so-called “saturation length” [8,9]; and c) the giant-dune size is limited by the depth of atmospheric boundary layer [10]. However, it is always difficult to distinguish between the ripples in an ordinary photograph and the dunes in a satellite image because both repeated patterns are very regular. The geometrical data of ripples, dunes and giant dunes around the world indicate that there exists a uniform height-spacing relationship for these aeolian bedforms (see [11] and references therein). In this letter, we attempt to explain this height-spacing law theoretically.

Although the form and scale of the actual bedform is governed by sand availability, vegetation cover, grain size distribution, wind velocity and directional variability etc. [4,12,13], the interaction between wind and sand is a fundamental physical process. In the equilibrium state of sand and wind, the bedform self-adjusts to a special shape. Here we propose the following “minimum force” hypothesis:

\textit{Given the sand mass and wind velocity, the bedform must tend to evolve to a shape subjected to the minimum force exerted by the wind.}

In fact, an analogous idea was put forth by Bosworth in 1922 while exploring the formation of Yardangs [14]. The coupling process of water and bottom to minimize friction was also noticed by Folk [15]. But, such a simple principle has been ignored for many decades.

Some researchers argued that the regular transverse or longitudinal vortices in the atmospheric boundary layer cause the instability of the sand bed [2,15]. The vortices really occur on the lee sides of mature ripples and dunes [1,16,17], although there is not enough field evidence to support this explanation for dune initiation [13]. The flow dynamics are more complicated than that depicted by Folk [15]. As shown in the turbulent flow over alluvial dunes by using numerical methods, the separated vortices are formed downstream of the dune crest due to Kelvin-Helmholtz instabilities and the “boil” vortex evolves in form of a hairpin vortex [18,19]. Figure 1, simulated by the $K-\varepsilon$ turbulent model in FLUENT Software, shows the typical vortices near the two-dimensional periodic dunes in the steady state. The flow pattern is insensitive to wind velocity and dune height. The Reynolds number $Re$ based on the wind velocity at crest and the dune height is about $1.37 \times 10^6$ in fig. 1. So the flow near the bed of a transverse aeolian bedform can be roughly replaced by a single row of vortices equally spaced and rotating in the same direction. The existence of the “unstable” single vortex row under this condition is reasonable [20].

The shear stress of airflow plays a dominant role in the sediment transport. If the shear stress between wind and bed is smaller than the threshold stress of grain movement, the size and shape of bedform will be stable. Obviously, the streamlined shape offers minimum wind resistance. For an
A profile of a streamline can be expressed as an infinite single row of vortices with separation $l$ and position $(nl, 0)$ where $n$ is an integer in inviscid fluid dynamics, the profile of a streamline can be expressed as

$$\cosh\left(2\pi \frac{y}{l}\right) - \cos\left(2\pi \frac{x}{l}\right) = C,$$

where $C$ is a constant.

Another important factor in the evolution of aeolian bedforms except wind is gravity which restricts the obliquity of the surface through the following mathematical relationship:

$$\left| \frac{dy}{dx} \right| \leq \tan \phi,$$

where $\phi$ is the angle of repose.

The upper limit of the height-spacing ratio $\lambda_{up}$ of transverse aeolian bedforms can be derived from eqs. (1) and (2), as listed in table 1. The maximum value of dune height in the Badain Jaran desert we measured by using the total station in 2007 is 525.10 m. The mean spacing of dunes and the angle of repose of slipfaces in that region is 3.575 km and 30°, respectively. The maximum height-spacing ratio $\lambda_{max}$ for these giant dunes is thus 0.147 smaller than $\lambda_{up}$ predicted by this simple model.

In this letter, only the transverse aeolian bedform in the dynamics equilibrium state was analyzed with great simplification. The obtained upper limit of height-spacing ratio is undoubtedly suitable for ripples, small-scale dunes and giant dunes. The “minimum force” hypothesis we proposed need to be further verified by taking into consideration of the interaction between bedform, airflow and sediment transport.

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