Influence of moisture on the entrainment of sand by wind

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Abstract

The theory of wet granular material is applied to the study of the influence of moisture on the entrainment of sand by wind in the first process of aeolian sand transport. The interparticle force due to water bridge is calculated using the toroidal approximation at first; and then the moment balance of a grain in the surface layer of sand bed is considered; finally, the change of threshold friction velocity with water content is obtained.

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1. Introduction

Aeolian erosion occurs only when a threshold value of the wind velocity is reached and this threshold depends on the features of a sandy bed surface. Among the several factors that govern threshold conditions, moisture content is one of the most significant because it contributes greatly to the binding forces that keep the particles together [1–3].

Although many studies were conducted to determine the influence of moisture on the entrainment of soil or sand particles by wind, its effect is still not well understood [4–7]. To date, numerous models [1,2,8–16] to predict the change of threshold friction velocity with moisture content have been developed. Most models are empirical (Chepil66 [8], Belly64 [9], HKKH84 [10], SF95 [12], SRL96 [13]) or semi-empirical (FMB99 [2]). The detailed descriptions of these models can be found in the review paper of Cornelis and Gabriels [3] and references therein. Here we pay more attention to the three typical theoretical models, namely MN89 [1], GD90 [11] and CGH04 [14–16]. The establishing procedures of them are analogous. Two critical points while modeling the entrainment of wet sands by wind are how to simplify the natural grain contact while water exist and how to calculate the interparticle force due to moisture. The grain contact was reasonably approximated by disymmetric cones in model MN89. This treatment leads to a non-dimensional geometric coefficient in the expression of interparticle force. As pointed out by Cornelis and Gabriels [3], MN89 is not practical because the geometric coefficient cannot be readily determined. In model CGH04, one principal radius of curvature of the air–water surface was supposed to be related to the third power of another and then the effects of grain shape were successfully eliminated. However, this assumption was not examined carefully. Although it seems that the contact of spherical grains as in model GD90 is simplest, direct contact seldom occurs under their hypothesis of grain shape because there are no absolutely smooth particles in practice. Once the mode of grain contact is founded, the next step every model cannot avoid is to determine the interparticle force due to moisture. Moisture is retained in sandy bed by two processes, water film and water bridge [1,4]. Water film may appear on the grain surface. Water bridge may form around the contact points of the grains. The contribution of water film to interparticle force is much lower than that of water bridge [17]. The water only appear on the grain surface in model GD90. The interparticle force due to water bridge, namely capillary force, is the sum of two components. One part is due to surface tension of the water at three-phrase contact line. The other is due to the pressure different across the water–air interface. Taking into account these two components and using respective simplifications of grain contact, the capillary force was all written as an analytical function of the pressure difference between the inside of water bridge and its outside in both MN89 and CGH04. A slight difference between two models is the coefficient before...
the negative power of pressure difference. But, the relation between the pressure difference and the moisture content was empirically obtained in both models. In other words, the interparticle force was not computed accurately. It should be noticed that the interparticle force in CGH04 we mentioned does not include Coulomb force and van der Waals force which were considered in their original derivations [16].

Sand is a glaring example of granular material which is of great interest in physicists. The purpose of this research is to calculate the interparticle force due to water bridge using the theory of wet granular material and then give a new theoretical model for the effect of moisture on the entrainment of sand by wind.

2. Interparticle force

The exact value of capillary force can only be obtained by solving numerically the Laplace–Young equation for the surface shape of water bridge. Many researches, e.g. [18–20], have shown that the error of toroidal approximation is very small. In the toroidal approximation, the force can be calculated either on the three-phase contact area [21] or on the gorge of the water bridge [22]. A more accurate approximation was given by Lian et al. [20] and it is therefore used here.

Fig. 1, in which θ and ψ are the contact angle and half-filling angle, gives the coordinates to describe the geometrical shape of a water bridge bonding two spherical monosized grains of radius \( r \) and separated by a distance \( 2x \). In the case of toroidal approximation, two dimensionless principle radii with respect to grain radius can be derived as

\[
\rho_1^* = \frac{\rho_1}{r} = \frac{x + 1 - \cos \psi}{\cos(\psi + \theta)}
\]

\[
\rho_2^* = \frac{\rho_2}{r} = \sin \psi + \rho_1^* \sin(\psi + \theta) - 1
\]

(1)

The water volume corresponding to one grain in Fig. 1 is

\[
\nu = \frac{V}{\pi r^3} = \left[ \rho_1^* + (\rho_1^* + \rho_2^*)^2 \right] \rho_1^* \cos(\theta + \psi)
\]

\[
- \frac{1}{3} \rho_1^* \cos^3(\theta + \psi) - \frac{1}{2} \rho_1^* (\rho_1^* + \rho_2^*)
\]

\[
\times [\sin(2\theta + 2\psi) + \pi - 2\theta - 2\psi]
\]

\[
- \frac{1}{3} (2 + \cos \psi)(1 - \cos \psi)^2
\]

\[
\text{(3)}
\]

The maximum value of dimensionless local mean meridian curvature is

\[
(H_F^*)_{\text{max}} = \frac{2 \sin \psi - \rho_1^* - \rho_2^*}{2 \rho_1^* \sin \psi}
\]

(4)

In the absence of gravitational effects of water bridge, the dimensionless interparticle force is

\[
f_c = \frac{F_c}{\pi r \gamma} = 2k_c \rho_1^* \left\{ 1 + k_c \rho_1^* [k_H(H_F^*)_{\text{max}} + k_H - 1] \right\}
\]

(5)

where \( \gamma \) is the surface tension, \( k_c \) is the ratio of the real radius to the approximate radius at the neck, \( k_H \) is the dimensionless mean curvature defined as \( (H^* + 1)/(H^*_{\text{max}} + 1) \) in which \( H^* \) is the real mean curvature.

On the base of rigorous numerical results, it was found that \( k_c \) and \( k_H \) are relatively insensitive to the bridge volume and can be empirically expressed in terms of separation

\[
k_c = 1.0 - 0.00032 \exp \left( \frac{6.8 x}{x_c} \right)
\]

(6)

\[
k_H = 0.91 - 0.10 \frac{x}{x_c} - 0.61 \left( \frac{x}{x_c} \right)^2
\]

(7)

where \( x_c \) is the dimensionless critical rupture separation distance. This parameter can be determined by strictly considering the minimization of free surface energy. Based on their numerical solutions, Lian et al. [20] gave the following simple expression

\[
x_c = 0.5(1 + 0.5 \theta) \sqrt{\pi \nu}
\]

(8)

The expression of interparticle force, Eq. (5), is applicable for separation distances up to \( x_c \) and for any bridge volume and contact angle.

3. Moisture content

The moisture content is defined as the ratio of the mass of water to the mass of sand grains

\[
w = \frac{m_W}{m_s} = \frac{n \rho_w V}{4 \rho_s \pi r^3 / 3} = \frac{3n \nu \rho_w}{4 \rho_s}
\]

(9)
where $\rho_w$, $\rho_s$ and $n$ are water density, sand density and the mean water bridge coordination number, respectively. The volume of one water bridge is given by Eq. (3).

The mean water bridge coordination number $n$ is a function of moisture content in nature. It was shown that no water bridges were observed for moisture contents below a critical value $w_c$; once above $w_c$, $n$ jumps to a saturation value $n_s$ rapidly [23]. For simplicity, $n$ is supposed to be a constant $n_s$ which is approximate to 6 for loose packing in the experiment of Kohonen et al. [23].

Given the moisture content $w$, the volume of water bridge $\nu$ can be determined by Eq. (9). Then, the half-filling angle $\psi$ is obtained by solving Eq. (3) for a given separation distance $\alpha$ and complete wetting ($\theta = 0^\circ$). Finally, the interparticle force $f_c$ as a function of moisture content $w$ is calculated by Eq. (5). As the water content in the granular material exceeds a critical value, the liquid bridge will form among three or more grains [24] and Eq. (5) will be inapplicable.

4. Threshold friction velocity

In wind tunnel experiments, the initiation of particle movement was detected by the naked eye [5], a small trap [9] or an acoustic sediment sensor [15]. The visual observe [5] is empirical entirely. The measurement of sand flux via a small trap [9] enlarges threshold friction velocities. Recording three impacts within one minute [15] is a better method. From a theoretical point of view, the model of three sands as described in Fig. 2 was subsequently employed to the entrainment of sand by wind under dry or wet conditions [1,4,14,16].

At the motion instant of grain A in Fig. 2, the balance of moment leads to

$$F_c|OQ| + mg|OR| = F_d|OP|$$  \(10\)

where only the main aerodynamic force $F_d$ is taken into account.

The gravity force is

$$mg = \frac{1}{6} \rho_s g \pi d^3$$  \(11\)

where $d$ is the diameter.

The drag force can be expressed as [4]

$$F_d = K_d \rho_s d^2 u_{tw}^2$$  \(12\)

where $K_d$ is treated as a dimensionless proportionality constant, $u_{tw}$ is the threshold friction velocity of wet sands.

Substituting Eqs. (5), (11) and (12) into Eq. (10), we obtain

$$u_{tw} = \sqrt{\frac{A \frac{K_d}{\rho_s d} + B \frac{\rho_s}{\rho_d} \frac{g d}{u_{tw}^2}}}$$  \(13\)

where $A$ and $B$ are two dimensionless coefficients. When $w=0$, Eq. (13) is just Bagnold’s threshold friction velocity for dry sands. The coefficient $B$ has been found to be between 0.01 and
0.04 [4]. \( B = 0.019 \) is adopted here. So, the threshold friction velocity for wet sands can be written in a more general form

\[
\begin{align*}
u_{tw} &= \sqrt{\frac{\gamma f_c}{\rho_s d} + \nu^2} \\
&= \sqrt{\frac{\gamma f_c}{\rho_s d} + \nu_{tw}^2}
\end{align*}
\tag{14}
\]

where \( \nu_{tw} \) is the threshold friction velocity under dry conditions.

5. Numerical result and discussion

The separation distance \( \alpha \) is an important parameter which has not been introduced in the previous theoretical studies of the threshold friction velocity. Its effect is shown in Fig. 3. The threshold friction velocity decreases as the separation distance between grains increases. Although the grains also do not touch each other in the schematic diagram of the model of Cornelis et al. (see Fig. 1(c) in Ref. [14]), the influence of the separation distance is not been taken into consideration actually. Values of \( \alpha \) in the range of 0.01–0.10 seem to correlate the experimental data well while modelling the tensile strength of wet granular materials [21]. The value \( \alpha = 0.06 \) is used in the following calculation.

Table 1 lists some widely used models. FMB99 is an empirical extension of MN89 to other soil types. The model of CGH04 is not included for two reasons. Firstly, its expression is very complicated. Secondly, its prediction is found to be in good agreement with an empirical model Chepil56. The parameters and coefficients in the expressions of models were often determined by wind tunnel experiments. For example, it was found that \( w_{1.5} = 0.013 \) in Chepil56, GD90 and SF95, \( a_1 = 6.12 \times 10^{-7} \text{ kg/s}^2 \), \( a_2 = 738.2 \text{ kg/m}^2 \text{s}^2 \) in GD90 and \( a = 1.2, b = 0.6 \) in FMB99, respectively. However, the wetted soils used in these tests were not in equilibrium with the atmospheric humidity and the measurement results were thus affected by considerable evaporation and soil drying in the course of test. This phenomenon has not been aware until the recent years [3,5,7].

The comparison between the model predictions and wind-tunnel data [3] is shown in Fig. 4. It seems that the tendencies of model Belly64, GD90 and FMB99 are in accord with experimental data in the whole range of observed moisture content, but obviously disaccord in quantity. For small moisture content \( w \leq 0.002 \), most models (e.g. Chepil56, HKK88, SF95 and SRL96) agree well with the experimental measurements. For large moisture content \( w \geq 0.010 \), the models of Chepil56, SF95 and SRL96 overestimate the values of \( \nu_{tw} \) greatly. The present model follows the data well than other models for \( w \geq 0.002 \).

6. Summary

In summary, the influence of moisture on the entrainment of sand by wind is studied theoretically and numerically. The interparticle force due to water bridge is considered using the toroidal approximation in the absence of gravitational effects. A new formula to predict the threshold friction velocity is presented. The current model plays well for the moisture content larger than 0.002 about.

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