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Creation of an ultra-long depth of focus super-resolution longitudinally polarized beam with a ternary optical element

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Abstract

We proposed a method to design a ternary optical element in order to achieve a needle of super-resolution longitudinally polarized beam with ultra-long depth of focus, and obtained a beam with a size of 0.3995\(\lambda\) and depth of focus of 12.83\(\lambda\) after focusing a ternary optical element modulated, radially polarized Bessel–Gaussian beam with an aplanatic lens of numerical aperture 0.95. The algorithm we used to design the ternary optical element is based on axial uniformity in the focal region, which allows rapid searching speeds and excellent performance. The ratio of pupil radius to the beam waist was set as 0.57, making the peak intensity of the incident beam occur at the rim of the lens aperture, which maximized the possible resolution of the focused beam.

Keywords: super-resolution, diffractive optical element, radially polarized light

(Some figures may appear in colour only in the online journal)

1. Introduction

Recently, research on longitudinally polarized laser beams has become a very hot topic due to its potential applications in various areas, such as particle acceleration [1], second-harmonic generation [2], fluorescent imaging [3], optical data storage [4] and optical trapping [5]. A number of methods have been proposed to produce a needle of subwavelength longitudinally polarized beam with long depth of focus (DOF). Wang \textit{et al} [6] used a phase plate to modulate an incident radially polarized Bessel–Gaussian (BG) beam. The light beam was focused by a high numerical aperture (NA) lens to produce a needle of longitudinally polarized light. Rajesh and Dehez \textit{et al} [7, 8] made use of a lens axicon or axicon to produce longitudinally polarized light. Kuang \textit{et al} [9] utilized a combination of amplitude and phase encoding to realize subwavelength focal spots and extend their DOF. Kitamura \textit{et al} [10] focused an annular beam of finite width to produce longitudinally polarized light.

It is of great importance to design the diffractive optical element (DOE) efficiently and accurately. Much work has been done to realize the desired goal [12–20, 24]. Among the methods of designing DOE, Huang \textit{et al} [11] formulated a method that combines the global-search-optimization (GSO) algorithm and the tight focusing properties of radially polarized light to design a DOE. Sheppard \textit{et al} [12] proposed an analytic solution based on the scalar paraxial approximation. Liu \textit{et al} [13] gave a method via traditional exhaustive searching methods in the paraxial Debye regime. Yun \textit{et al} [14] and Zhang \textit{et al} [15] presented a method through solving the nonlinear equations, but this method requires an estimated value and cannot ensure that we obtain...
the global optimal solution. Yu et al [16] made use of the constraint simulation annealing algorithm and got good results, but the computational cost is large. In this work, we proposed an algorithm to design the ternary optical element based on rigorous vectorial diffraction theory. In this algorithm, the exhaustive searching algorithm was combined with the restrictions of the axial uniformity and transversal spot size, which allows rapid searching speeds and excellent performance. The ternary optical element was designed with three different amplitude transmissions (1, 0, −1), and a needle beam has a transverse size of 0.3995λ and depth of focus of 12.83λ after focusing a ternary optical element modulated radially polarized BG beam with an aplanatic lens of NA 0.95. The rest of the paper is organized as follows. In section 2, the design principle and method are given. In section 3, numerical calculation and simulation are presented. Finally, the work is concluded in section 4.

2. The design principle and method

A schematic of the proposed system is shown in figure 1. A radially polarized BG beam is incident on a ternary optical element, and then is focused by a lens with an NA of 0.95, as shown in figure 1(a). The ternary optical element is presented in figure 1(b) and consists of one zero transmission belt in the central area (marked in black) and four belts with transmissions of −1 and +1 alternately in the outer region of the aperture. The yellow color represents transmission of −1 and the white color represents the transmission of +1. The central block area makes the incident beam become an annular beam, which means that the portion of high frequency band increases, the focus spot becomes small and the DOF increases [21]. The four belts in the outer region are a compromise between the longitudinal field strength and the optical efficiency. We know that the more belts there are, the greater is the portion of the longitudinal field component, but the optical efficiency drops with an increase in the number of the belts. Here, optical efficiency is defined as the ratio of the total energy within the focal volume after conversion to its original energy within the focal volume before conversion [6]. In the focal region of the lens, a super-resolution focal spot with a uniform size and an ultra-long DOF is obtained. The angle θi (i = 1–4) corresponds to the aperture half angle of the ith belt and is computed as θi = sin−1(riNA), where ri is the normalized radius of the ith belt.

The optical characteristics of the schematic in figure 1 may be calculated and analyzed. The electric field distribution in the vicinity of the focal region can be obtained using Richard and Wolf’s theory [22], which is expressed as [23]

\[
E_x(r, z) = A \int_0^\alpha \sqrt{\cos \theta} \sin(2l(\theta)) J_1(kr \sin \theta) \exp(ikz \cos \theta) \, d\theta
\]

where \(A = 1, k = 2\pi/\lambda\) is the wavevector, \(\alpha = \sin^{-1}(NA/n)\) is the maximum aperture half angle and \(n\) is the refractive index of the propagation medium. In the present study, light travels in free space. Therefore, \(n = 1\). \(J_0(x)\) and \(J_1(x)\) denote the Bessel function of the zero and first orders, respectively. The function of the radially polarized BG [23] beam is

\[
l(\theta) = J_1 \left( \frac{2\beta \sin \theta}{\sin \alpha} \right) \exp \left[ - \left( \frac{\beta \sin \theta}{\sin \alpha} \right)^2 \right]
\]

where \(\beta\) is the ratio of the pupil radius to the beam waist. We investigate the dependence of DOF and full width at half maximum (FWHM) on the \(\beta\) in clear aperture focusing conditions. As shown in figure 2. The FWHM increases and the DOF first decreases slightly and then increases as \(\beta\).
increases. The DOF is defined as the two-point distance where the intensity maximum $I_{\text{max}}$ along the $z$-axis decreases to 0.8$I_{\text{max}}$ and $\beta = 0.57$ corresponds to the peak intensity of the radially polarized BG beam at the rim of the focusing lens. Figure 2 shows that selection of $\beta = 0.57$ is a reasonable compromise between optical efficiency and the resolution of the focused beam, further decrease of $\beta$ would increase the resolution a little. However, more light would be outside the aperture of the focusing lens. The effect of the ternary optical element on the focusing system can be calculated by replacing $l(\theta)$ with $l(\theta)T(\theta)$ in equation (1), where $T(\theta)$ is the transmission function of the ternary optical element and is expressed as

$$T(\theta) = \begin{cases} 
0 & 0 \leq \theta < \theta_1 \\
-1 & \theta_1 \leq \theta < \theta_2, \quad \theta_3 \leq \theta < \theta_4 \\
1 & \theta_2 \leq \theta < \theta_3, \quad \theta_4 \leq \theta < \alpha 
\end{cases}$$

where $\theta$ is the aperture half angle.

In the optimization process, we divide it into two steps. Firstly, we use an exhaustive searching algorithm to obtain a certain number of potential solutions under the restriction of uniformity of the beam along the $z$-axis in the focal region and the FWHM of the focal spot, where we restrict them to be below 0.05 and 0.41$\lambda$, respectively. The combination of the two restrictions allows a fast searching speed. The uniformity along the $z$-axis is defined as the ratio of the intensity difference to the summation between the maximum and minimum intensity in the range from $-1.5\lambda$ to $1.5\lambda$ along the $z$-axis. The search space is placed in the range of normalized radii ($r_1$, $r_2$, $r_3$, $r_4$) with a tolerance of $\Delta r = 0.01$. The starting point for $r_1$ was 0.5. Our computing environment was the Matlab R2012a UNIX, which runs on an eight-core parallel computing system. Optimized results are obtained in 0.6 h, which is much faster than the traditional exhaustive searching algorithm based on FWHM and DOF searching criteria. Then, the FWHM is sorted from small to large and some favorable results are selected. In order to obtain an ultra-long and uniform subwavelength light beam, the two decimal digits of the tolerance are extended to four decimal digits manually according to experience and when it does not take too much time. Finally, we can find some good result. The most important feature of our optimization method is time saving.

### 3. Design results and analysis

The optimized normalization radii are obtained as follows: $r_1 = 0.8875$, $r_2 = 0.8965$, $r_3 = 0.9191$, $r_4 = 0.9481$ and $r_5 = 1$. Considering the rotational symmetry the electric energy density distributions of the radial, longitudinal and total fields on the $y$-$z$ plane are presented in figure 3. Figure 3(a) shows that the axial electric field energy density is very uniform. The beam has a size of 0.3995$\lambda$, and it propagates without divergence over 12.83$\lambda$, which is a highly localized super-resolution non-diffraction beam. The extended DOF is almost 15 times greater than that generated with a clear aperture. Compared with the customarily used axial extent of the ordinary focus, which is defined as $\lambda/\text{NA}^2$ (for $\text{NA} = 0.95$ it gives value 1.11$\lambda$) one can find that the obtained DOF of 12.83$\lambda$ in this work is extended to about 11.56 times. The electric energy density profile of the transversal behavior of this beam on the focal plane is shown in figure 3(d). The FWHM of the total electric density profile is 0.3995$\lambda$, which is smaller than that of 0.58$\lambda$ obtained through direct focusing of the BG beam with $\beta = 0.57$. The area of the super-resolution spot is 0.126$\lambda^2$. As a longitudinally polarized beam, the maximum electric density of the transversal polarization is only 5% of that of the longitudinal electric density. Another way to evaluate the quality of this beam is to look at the energy contained in longitudinal polarization in the focused beam, which is defined as the figure of merit as

$$\eta = \frac{\Phi_z}{\Phi_z + \Phi_r} \tag{4}$$

with

$$\frac{\Phi_z}{\Phi_z + \Phi_r} = 2\pi \int_{r_0}^{r_0} |E_z(r,0)|^2 r dr, \quad \frac{\Phi_r}{\Phi_z + \Phi_r} = 2\pi \int_{r_0}^{r_0} |E_r(r,0)|^2 r dr \tag{5}$$

where $r_0$ is the first zero point in the distribution of radial electric intensity, which is 0.65$\lambda$, as shown in the $|E_z|^2$ curve of figure 4. The figure of merit of the beam we generated is $\eta = 0.876$. The beam performance of our method is compared with those generated with other methods, as shown in table 1.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\beta$</th>
<th>FWHM ($\lambda$) at 0.8$\text{NA}^2$</th>
<th>DOF ($\lambda$)</th>
<th>Beam quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear aperture</td>
<td>0.57</td>
<td>0.586</td>
<td>0.86</td>
<td>0.434</td>
</tr>
<tr>
<td>Center block</td>
<td>0.57</td>
<td>0.427</td>
<td>2.296</td>
<td>0.81</td>
</tr>
<tr>
<td>with $r = 0.8875$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reference [7]</td>
<td>0.43</td>
<td>~4</td>
<td>0.81</td>
<td></td>
</tr>
<tr>
<td>Reference [10]</td>
<td>0.4</td>
<td>~7</td>
<td>0.858</td>
<td></td>
</tr>
<tr>
<td>Reference [8]</td>
<td>0.395</td>
<td>~6</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>Our method</td>
<td>0.57</td>
<td>0.3995</td>
<td>12.83</td>
<td>0.876</td>
</tr>
</tbody>
</table>

As described in table 1, in clear aperture conditions, the FWHM is 0.586$\lambda$ and the parasitic radial component accounts for 56.6% of the total electric energy. When the block at the center has a normalized radius of 0.8875, the FWHM is reduced to 0.427$\lambda$ and the longitudinal component is increased to 81% of the total electric energy. However, the DOF is not significantly extended. Reference [7] used a phase plate to modulate the incident radially polarized BG beam without blocking the central area. Reference [10] utilized a combination of amplitude phase encoding, cylindrical polarization and dual beam to realize subwavelength focal spots and extended DOFs, where the FWHM is 0.4$\lambda$, and the DOF and beam quality are 7$\lambda$ and 0.858, respectively. Reference [8] combined the lens axicon with a binary phase plate. The FWHM was a little smaller; the DOF was only half of our method and the beam quality was also lower. In our method, we used a block area identical to the second case of the central block in table 1. Considering the phase modulation relation $\lambda = \frac{\partial I}{\partial \theta}$ and the intensity difference to the summation between the maximum and minimum intensity in the range from $-1.5\lambda$ to $1.5\lambda$ along the $z$-axis.
in the outer area, the FWHM, DOF and beam quality were better than most of the reported values in the literature. This may be due to lots of reasons. First, a combination of phase modulation and the central block is adopted. The central block squeezed the focal spot to a certain extent and elongated the DOF, while the phase modulation in the outer region narrowed the focal spot further and elongated the DOF noticeably. Second, the four belts are adopted in the outer region, which enhanced the longitudinal field intensity in the focal region [11]. Third, the parameter $\beta$ of the incident radially polarized BG beam is 0.57, which suggests that the peak intensity of the incident beam is at the rim of the lens.

Finally, we give the error analysis when the radii of the ternary optical element deviate from their original values. We let the normalized radius of each zone deviate within radius error tolerance $(-0.005, 0.005)$ and give the dependence of the difference between current FWHM, DOF and uniformity along the $z$-axis ($-5\lambda$, $5\lambda$) and original values on radius tolerance. Figure 4 shows the dependence of the FWHM difference on each radius $r_1, r_2, r_3, r_4$ respectively. We can see from figures 4(a) and (c) that FWHM increases as the tolerance of $r_1$ and $r_3$ increases, while figures 4(b) and (d) show that FWHM decreases as the tolerance of $r_2$ and $r_4$ increases. It is obvious that the FWHM difference is less than 0.01$\lambda$, thus the FWHM difference is almost negligible in the radius tolerance range from $-0.005$ to $0.005$.

The dependence of differences of DOF and uniformity along the $z$-axis ($-5\lambda$, $5\lambda$) on radius tolerance can be found from figure 5. According to figure 5, the profiles of the corresponding curves are almost the same. The profiles of the dependence of uniformity on tolerance $r_1, r_2, r_3, r_4$ show a ‘V’ symbol, which means the original radii have the best uniformity. The uniformity becomes worse and worse as the radii deviate from the original value. One can see from figures 5(a)–(c) that the original radii have the largest DOF, when the radii deviate from the original value the DOF becomes small. In figure 5(d) an exception occurs, when the tolerance is within $(0, 0.002)$, the DOF slightly increases with the tolerance and the uniformity becomes a little worse, but this is acceptable. From the analysis above, we let one of the radii deviate from the original value within tolerance $(-0.005, 0.005)$ and the remainder are unchanged; we can make a coarse judgment on what would happen if the radii deviate from their original values.

4. Conclusion

In conclusion, we proposed an algorithm to design a ternary optical element based on rigorous vectorial diffraction theory.
Figure 4. The dependence of FWHM difference on the tolerance of (a) $r_1$, (b) $r_2$, (c) $r_3$, (d) $r_4$.

Figure 5. The dependence of uniformity and DOF difference on the tolerance of (a) $r_1$, (b) $r_2$, (c) $r_3$, (d) $r_4$.

In this algorithm, the exhaustive searching algorithm was combined with the restrictions of the axial uniformity and transversal spot size, which allows rapid searching speeds and excellent performance. Through the algorithm, a needle of super-resolution longitudinally polarized beam with ultra-long depth of focus was obtained. The beam
had a spot size of 0.3995λ and depth of focus of 12.83λ after focusing a ternary optical element modulated, radially polarized Bessel–Gaussian beam with an aplanatic lens of numerical aperture 0.95.

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