Dispersion-free quantum clock synchronization via fiber link

Fei-yan Hou a, Rui-fang Dong a,* , Run-ai Quan a, Yu Zhang a,b, Yun Bai a,b, Tao Liu a, Shou-gang Zhang a, Tong-yi Zhang c

a Key Laboratory of Time and Frequency Primary Standards, National Time Service Center, Chinese Academy of Sciences, Xian 710600, China
b Graduate University of Chinese Academy of Sciences, Beijing 100039, China
c State Key Laboratory of Transient Optics and Photonics, Xi'an Institute of Optics and Precision Mechanics, Chinese Academy of Sciences, Xian 710119, China

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Abstract

We proposed a fiber-based quantum clock synchronization protocol by employing the dispersion cancellation feature of the frequency anti-correlated entangled source under the quantum interference measurement. It is shown that, the accuracy of the synchronization is mainly dependent on the bandwidth of the frequency entangled biphoton spectrum and on the temperature variation induced fluctuations in both the reference fiber coiling and the fiber link. With this proposal, synchronization between two clocks at a distance of ten kilometers can be implemented with an accuracy below a picosecond.

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1. Introduction

High-accuracy synchronization of spatially separated atomic clocks plays an important role in fundamental physics and in a wide range of applications such as communications, message encryption, navigation, geolocation and homeland security. Einstein synchronization is the most applicable method for clock synchronization which is implemented by an exchange of electromagnetic pulses between two spatially separated clocks and measuring their time of arrival (Einstein, 1905). Based on this method, atomic clocks are compared by means of the satellite-based Global Positioning System (GPS) (Parkinson and Spilker 1996; Kaplan 1996; Hofmann-Wellenhof et al., 1993), and by employing telecommunication satellites for two-way satellite time and frequency transfer (TWSTFT) (Kirchner, 1999; Bauch et al., 2009). The time scales can be synchronized with an uncertainty up to 1 nanosecond. However, such accuracy is far from enough to satisfy the growing requirement for comparison of the new generation of high-precision atomic clocks, meanwhile limits many other prospective applications, such as coherent detection of electromagnetic signals and fundamental physics principle tests. In a recent future, the ACES mission (Atomic Clock Ensemble in Space) based on an elaborated manipulation of radiofrequency signals will demonstrate an improved accuracy as low as 100 ps (Cacciapuoti and Salomon, 2009). To further improve the synchronization accuracy, the time transfer by laser link (T2L2) was proposed and applied in the real satellite-based synchronization system (Fridelance and Veillet, 1995; Fridelance et al., 1997; Guillemot et al., 2008; Vrancken, 2008). With a better control of light pulses than radio wave propagation, the recent T2L2 experiment gives the best clock synchronization accuracy within 50 ps (Guillemot et al., 2008; Vrancken, 2008), though it has a strong dependence on the weather condition. Owing to the low loss, strong anti-jamming ability, high transfer security, and wider transmission...
bandwidth of optical fiber, time transfer through optical fibers has demonstrated a further reduction of uncertainty (Czubla et al., 2006; Sliwczynski et al., 2010). Therefore, synchronization via optical fibers becomes a promising candidate as a part of the infrastructure connecting the ground station setups during forthcoming T2L2 and ACES experiments, supporting deep-space network and antenna arrays in astronomy (Calhoun et al., 2007) or in accelerators for particle physics (Kartner et al., 2010).

One of the most important characteristics among the above mentioned methods is the uncertainty evaluation of atomic clocks and time scales. Upon all kinds of noises contributing to the achievable accuracy, the noise resulting from the time-of-arrival measurement is the fundamental noise and is classically limited by the available power and bandwidth, which is the so called shot noise limit. For example, according to the settings of the T2L2 instrument on board the Jason-2 space vehicle, this limit contributes an accuracy of about 3.5 ps over an integration time of 100 s. Quantum clock synchronization is a new technology which utilizes quantum sources and quantum measurement technique in the clock synchronization system to break through the shot noise limit on the classical system, finally reaches the fundamental limit for quantum mechanics, that is, the Heisenberg limit. Since it was proposed in the beginning of this century (Giovannetti et al., 2001a), quantum clock synchronization has attracted much research interests (Giovannetti et al., 2002; Giovannetti et al., 2001b; Giovannetti et al., 2004; Valencia et al., 2004; Bahder and Goldberg, 2004). According to recent investigations, quantum clock synchronization, as a highly promising technology to improve the classical synchronization accuracy dramatically, will not only be capable of quantum encrypted (Giovannetti et al., 2002), but also have the ability of cancelling propagation noise induced by dispersion in the optical path (Giovannetti et al., 2001b; Giovannetti et al., 2004).

As light pulses will be broadened by the group velocity dispersion (GVD) after passing through dispersive medium, dispersion is one of the main restrictions to improve the accuracy of the fiber-link based clock synchronization systems. In this paper we propose a fiber-based quantum clock synchronization scheme to eliminate the timing error due to the GVD in the fiber, based on the quantum feature that the dispersion experienced by the frequency anticorrelated entangled source can be cancelled by the quantum interference measurement in the frame of reference of a Hong-Ou-Mandel (HOM) interferometer (Bahder and Goldberg, 2004; Hong et al., 1987; Steinberg et al., 1992a,b). Through a thorough analysis, we show that the final synchronization accuracy is jointly determined by the bandwidth of the frequency entangled biphoton spectrum as well as the temperature variation induced fluctuations in both the reference fiber coiling and the fiber link. The rest of the paper is organized as follows. Section 2 is a brief description of the scheme. Section 3 gives the theoretical derivation. Section 4 is the conclusion.

2. Scheme description

The dispersion-free fiber-link based quantum clock synchronization scheme is sketched in Fig. 1. Positions of clock A and clock B are linked by a fiber with an unknown length d. To synchronize clock A and clock B, the following procedures are implemented.

At clock A position, frequency anti-correlated photon pairs are generated by spontaneous parametric down-conversion (SPDC) in a nonlinear crystal (Louisell et al., 1961; Mollow, 1973; Rubin et al., 1994). In the SPDC process, a photon from an intense cw pump beam is occasionally annihilated due to the nonlinear interaction inside the crystal and two photons with lower frequencies, named as signal and idler, are created. The signal and idler photons are then departed spatially and disseminated through different fiber paths. The signal is traveled from A to B and then reflected back through the fiber link, while the idler is held at A and sent through the reference fiber coiling with an adjustable length of l0. After traveling through different fiber paths, the signal and idler photons are interfered at a 50/50 beam splitter. The two outputs of the beam splitter are subsequently detected by two single-photon counters D1 and D2. The photon count outputs are then sent into a high-speed AND-logic circuit for coincidence measurement. Such measurement is the so-called Hong-Ou-Mandel (HOM) interferometric measurement (Hong et al., 1987), which is also named as second-order quantum interference measurement. The length of the reference fiber coiling l0 is adjusted until the interferometer is balanced, which corresponds to the minimum coincidence. Once the HOM interferometer is balanced, the time delay in the fiber coiling l0 will be equal to the round-trip time delay in the fiber link 2d, that is, $b_1'2d = b_2' l_0$, where $b_1'$ and $b_2'$ denote the inverse group velocities in the fiber link and the adjustable fiber coiling, respectively. The achievable accuracy of such balance is then determined by the resolution of the HOM interferometer, which is shown limited by the spectral bandwidth of the entangled photons and unaffected by group velocity dispersion in the fiber (for details see Section 3). Since the adjustable fiber coiling l0 is in the laboratory, standard techniques can be applied to determine the delay in the fiber coiling accurately and, hence, the delay in the fiber link.

Maintaining the balance of the HOM interferometer, clocks A and B are used to record the SPDC photons creation times at position A $\{t_A^{(i)}\}$, and the arrival times of the signal photons at position B $\{t_B^{(i)}\}$ after traveling through the fiber link, where $i = 1...N$ denotes a series of N photon pairs. The time data are brought together through a classical communication channel for comparison, and the registration time difference $t_B - t_A$ is obtained by maximum “coincidences” of the photon registration time records. When the path length d is known, the measurement resolution is determined by the natural width of $G^{(2)}$ for SPDC (Valencia et al., 2004), which is given by the Fourier transform of the
biphoton spectrum and typically on the order of a few femtoseconds to hundreds of femtoseconds. Assume the real time difference between clock A and B is $t_0$, it is then estimated by $t_0 = (t_B - t_A) - \beta_1 d$ (Valencia et al., 2004). Informed this time offset, the clocks are then synchronized accordingly.

3. Theoretical analysis

To investigate the achievable accuracy of the above fiber-linked clocks synchronization scheme, the theoretical analysis is presented as follows.

Consider a cw pump at the frequency of $\omega_p$ and the degenerate phase matching such that the center frequencies of the SPDC generated signal and idler both equal half the pump frequency. This system produces a biphoton source of the form

$$|\psi\rangle = \int d\omega f(\omega) \hat{a}_s^\dagger \left( \frac{\omega_p}{2} + \omega' \right) \hat{a}_i^\dagger \left( \frac{\omega_p}{2} - \omega' \right) |0\rangle,$$

where $\hat{a}_s^\dagger$ ($\hat{a}_i^\dagger$) and $\hat{a}_i^\dagger$ ($\hat{a}_s^\dagger$) are the creation operators for the signal and idler, respectively; $|0\rangle$ is the vacuum state. $|f(\omega')|^2$ is the frequency spectrum of the biphoton state. As a signal photon at frequency $\omega_p/2 + \omega'$ is accompanied by an idler photon at frequency $\omega_p/2 - \omega'$, the signal and idler are frequency anti-correlated entangled.

The signal and idler photons are then separated and traveled through the fiber link and the adjustable reference fiber coiling, respectively. Due to the frequency dependence of the refractive index $n(\omega)$ in optical fibers, chromatic dispersion is induced during propagation of the signal and idler photons. This effect is described by the mode-propagation constant $\beta(\omega)$, which can be expanded as Taylor series about the frequency $\omega_0 = \omega_p/2$ (Agrawal, 2001)

$$\beta(\omega) = \frac{n(\omega)\omega}{c} = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2!} \beta_2(\omega - \omega_0)^2 + \cdots,$$

where $c$ is the velocity of light in vacuum, $\beta_m$ is defined as

$$\beta_m = \left( \frac{d^m \beta}{d\omega^m} \right)_{\omega=\omega_0} \quad \text{with} \ m = 1, 2, 3, \ldots$$

The first order term $\beta_1$ can be interpreted as the inverse of the group velocity $v_g$, the speed at which the pulse envelop travels (Agrawal, 2001):

$$\beta_1 = \frac{1}{c} \left( n + \omega \frac{dn}{d\omega} \right) = \frac{1}{c} \left( n - \frac{\lambda}{c} \frac{d\omega}{d\lambda} \right) = \frac{n_g}{c} = \frac{1}{v_g},$$

where $\lambda$ is the wavelength corresponding to $\omega$, and $n_g$ is the group refractive index.

The second order term describes the frequency dependence of the propagation speed (Agrawal, 2001):

$$\beta_2 = \frac{1}{c} \left( \frac{2}{d\omega} + \omega \frac{d^2\omega}{d\omega^2} \right) \approx \frac{\omega}{c} \frac{d^2\omega}{d\omega^2} \approx - \frac{\lambda^2}{2\pi^2} \frac{d\omega}{d\lambda^2} = - \frac{\lambda^2}{2\pi^2} D,$$

which represents the dispersion of the group velocity and is responsible for pulse broadening or distortion. This phenomenon is known as group velocity dispersion (GVD), and $D = - \frac{\lambda^2}{2\pi^2} \beta_2$ is the GVD parameter. $\beta_1$ and $\beta_2$ are normally specified by manufacturer of the fiber.

After passing through the fiber link $d$ and the fiber coiling $l_0$ respectively, the signal and idler photons are then interfered on a 50/50 beam splitter, as shown in Fig. 1. The annihilation operators for the two outputs 1 and 2 of the beam splitter are related to those for the signal and idler by the following formula:
\[ \hat{a}_1(\omega) = \frac{1}{\sqrt{2}} \hat{a}_i(\omega)e^{i\delta(\omega)2d} + \frac{1}{\sqrt{2}} \hat{a}_i(\omega)e^{i\delta(\omega)l_0}, \]
\[ \hat{a}_2(\omega) = \frac{1}{\sqrt{2}} \hat{a}_i(\omega)e^{i\delta(\omega)l_0} + \frac{1}{\sqrt{2}} \hat{a}_i(\omega)e^{i\delta(\omega)2d}, \]
where the propagation constants of the fiber link \( d \) and the fiber coiling \( l_0 \) are denoted as \( \beta^e \) and \( \beta^f \), respectively. Due to the Heisenberg uncertainty relation, these operators obey the canonical commutation relations:
\[ [\hat{a}_j(\omega), \hat{a}_k^+(\omega)] = 0, \]
\[ [\hat{a}_j^+(\omega), \hat{a}_k^+(\omega)] = 0, \]
\[ [\hat{a}_j(\omega), \hat{a}_k(\omega)] = \delta_{jk}(\omega - \omega_k), \]
\[ j, k = 1, 2, s, i. \]

According to the Mandel formula for detection (Hong et al., 1987), the coincidence rate at the photodetectors is given as follows:
\[ P_C = \int_0^T dt \int_0^T dt' \langle \Psi | E_1^+(t_1)E_2^+(t_2)E_1^+(t_1)|\Psi \rangle, \]
where \( T \) is the integration time window of the detectors. Omitting the irrelevant normalization constants, the positive- and negative-frequency field operators at detector \( j \) are defined by
\[ E_j^+(t_j) = \int d\omega \hat{a}_j^+(\omega)e^{i\omega t_j}, \]
\[ E_j^-(t_j) = \int d\omega \hat{a}_j^-(\omega)e^{i\omega t_j}, \text{ for } j = 1, 2. \]

Substituting (9) and (6) into (8) and taking the limit \( T \to \infty \) without loss of generality, one obtains
\[ P_C \propto \int d\omega \int d\omega' |\psi(\omega, \omega')\rangle^2 \]
\[ \times \int d\omega_2 (\langle \psi(\omega_1, \omega)\rangle \langle \psi(\omega_2, \omega)\rangle \langle \psi(\omega, \omega_1)\rangle |\Psi \rangle. \]

Using Eq. (1)–(5) and neglecting the dispersion terms higher than second order dispersion, we evaluate the integrand as below
\[ \langle \psi(\omega_1, \omega_2, \omega_2)\rangle^2 \]
\[ = |\langle 0|\hat{a}_1(\omega_1)\hat{a}_2(\omega_2)|\psi \rangle|^2 \]
\[ = \left\{ \frac{1}{2} |\hat{a}_1(\omega_1)\hat{a}_2(\omega_2)e^{i\delta(\omega_1, \omega_2)l_0} + e^{i\delta(\omega_1, \omega_2)2d} - \hat{a}_1(\omega_1)\hat{a}_2(\omega_2)e^{i\delta(\omega_1, \omega_2)l_0} + e^{i\delta(\omega_1, \omega_2)2d} |\Psi \rangle \right\}^2 \]
\[ = \left\{ \frac{1}{2} \delta(\omega_p - \omega_1 - \omega_2) f^{(\omega_p, \omega)} - \frac{1}{2} \delta(\omega_p - \omega_1 - \omega_2) f^{(\omega_p, \omega)} \right\}^2 \]
\[ \times \left( 1 - \text{Re} \left[ e^{-i\delta(\omega_p - \beta_1^e l_0 + \beta_2^f 2d)(\omega_1 - \omega_2)} \right] \right), \]
where
\[ f^{(\omega_p, \omega)} = \frac{1}{2} |\hat{a}_1(\omega_1)\hat{a}_2(\omega_2)e^{i\delta(\omega_1, \omega_2)l_0} + e^{i\delta(\omega_1, \omega_2)2d} - \hat{a}_1(\omega_1)\hat{a}_2(\omega_2)e^{i\delta(\omega_1, \omega_2)l_0} + e^{i\delta(\omega_1, \omega_2)2d} |\Psi \rangle \right\}^2 \]
\[ = \frac{1}{2} \delta(\omega_p - \omega_1 - \omega_2) f^{(\omega_p, \omega)} - \frac{1}{2} \delta(\omega_p - \omega_1 - \omega_2) f^{(\omega_p, \omega)} \times \left( 1 - \text{Re} \left[ e^{-i\delta(\omega_p - \beta_1^e l_0 + \beta_2^f 2d)(\omega_1 - \omega_2)} \right] \right).

We substitute (11) into (10) and drop the overall numerical factors, the final coincidence rate is yielded
\[ P_C \propto \int d\omega f^{(\omega, \omega)} \left\{ 1 - \cos[2\omega(\beta_1^e l_0 - \beta_2^f 2d)] \right\} \]
where $t_{A0}$ and $t_{B0}$ denote the times at which Clock A and B start their clocks, $t_0 = t_{B0} - t_{A0}$ is thus the time offset between these clocks.

Substituting (15)–(23) and neglecting the dispersion terms higher than the second-order dispersion term, one gets

$$G^{(2)}(t_B - t_A) = \left| \int d\omega' f(\omega') e^{i\omega t_0} e^{-i\omega t_A} \right|^2 = \left| \int d\omega' f(\omega') e^{i\omega t_0} e^{-i\omega t_A} \right|^2,$$

where $\tau = (t_B - t_A) - t_0 - \beta_1 d$. When the far-field condition is satisfied, which is reasonable in most application cases, the second-order correlation function is approximated to:

$$G^{(2)}(t_B - t_A) \sim f \left( \omega' = \frac{\tau}{\beta_1 d} \right)^2 \sim e^{-\frac{\tau^2}{\beta_1^2 d^2}}.$$

(17)

One can see that, it has the same shape as the Fourier transform of the SPDC spectrum, only broadened by a factor of $\Delta \omega^2 \beta_1 d$. Since $d$ is well measured by $d \approx l_0/2$ based on the HOM setup, and $\beta_1$ is normally specified by manufacturer of the fiber, the measurement of $t_B - t_A$ can reach, in principle, the same order of resolution as the natural width of $G^{(2)}$ for SPDC (Valencia et al., 2004)

$$\Delta(t_B - t_A) \sim 1/\Delta \omega.$$

(18)

The accuracy of the time difference between clock A and B $t_0$ is then given by

$$\Delta t_0 = \sqrt{\Delta^2(t_B - t_A) + \Delta^2(t_B' d)} = \sqrt{\frac{17}{16 \Delta \omega^2 x}}.$$  

(19)

It follows from Eq. (19) that, the accuracy of the scheme is dependent only on the bandwidth $\Delta \omega$ of the twin beam. Suppose that the bandwidth is $\Delta \omega \approx 10^{13}$ Hz, the accuracy is given by $\Delta t_0 \approx 0.1$ ps.

However, the propagation delay in the fiber is fluctuating, i.e., $\Delta t_{00} \neq 0$. Among various factors, the uncertainty of the propagation delay in the fiber is mainly caused by the fiber temperature variation and can be expressed as

$$\Delta t_{00} = \Delta t_{T_0} (\beta_1 t_0) = l_0 \frac{\partial \beta_1'}{\partial T} \Delta T_{t_0} + \beta_1' \frac{\partial l_0}{\partial T} \Delta T_{t_0},$$

(20)

where $\Delta T_{t_0}$ denotes the temperature variation of the fiber coating, the first right-hand-side term concerns the temperature dependence of the group velocity, and the second one the fiber thermal expansion. The two terms describe the thermal sensitivity of the fiber coating propagation delay. Such fluctuation contributes an error to the determination of the delay in the fiber link $\Delta t_d$

$$\Delta t_d = \sqrt{\left( \frac{1}{4 \Delta \omega} \right)^2 + \left[ \frac{1}{2} \Delta t_{T_0} (\beta_1 t_0) \right]^2}.$$  

(21)

Likewise, the measurement of $t_B - t_A$ is also affected by the temperature variation of the fiber link between clocks A and B, which can be given by

$$\Delta(t_B - t_A) = \sqrt{\left( \frac{1}{\Delta \omega} \right)^2 + \left[ \Delta \omega \cdot \Delta T_{T_0} (\beta_1' d) \right]^2}$$

$$= \sqrt{\left( \frac{1}{\Delta \omega} \right)^2 + \left[ \Delta \lambda \cdot \Delta T_{T_0} (D' d) \right]^2},$$

(22)

where $\Delta T_{T_0}$ denotes the temperature variation of the fiber link, $\Delta \lambda$ is the corresponding wavelength bandwidth of the bichromatic spectrum.

Go back to Eq. (19), the error in determining the time offset of clock A and B is then given by

$$\Delta t_0 = \sqrt{\Delta^2(t_B - t_A) + \Delta^2 t_d}$$

$$\approx \sqrt{\frac{17}{16 \Delta \omega^2} + \Delta^2 t_{00} (\beta_1' t_0^2) + \left[ \Delta \omega \cdot \Delta T_{T_0} (D' d) \right]^2}.$$  

(23)

Suppose that the twin beam is centered at $1.55 \mu m$, the bandwidth of $\Delta \omega \approx 10^{13}$ Hz corresponds to $\Delta \lambda \approx 13$ nm. According to Ref. (Sliwczynski et al., 2010), the thermal sensitivity of the fiber propagation delay in $1.55 \mu m$ transmission window is around $37$ ps/($K \cdot km$). Suppose that the fiber coating length in our scheme is $l_0 = 20$ km, and its temperature variation $\Delta T_{t_0}$ is controlled within $1 \text{mK}$, the uncertainty of the propagation delay in the fiber coating will be $\Delta t_{00} \approx 0.74$ ps. Substituting it to Eq. (21), one gets $\Delta t_d \approx 0.37$ ps.

In single-mode fibers, the thermal sensitivity of the GVD parameter can be taken as $1.5 \times 10^{-3}$ ps/($K \cdot km$) (Sliwczynski et al., 2010). For the fiber link with the length of $d \approx l_0/2 \approx 10$ km and the temperature fluctuation of $2 K$, the determination error of $t_B - t_A$ caused by the temperature fluctuation is $\Delta \lambda \cdot \Delta T_{T_0} (D' d) \approx 0.39$ ps. Adding this to Eq. (22), the total accuracy of the measurement of $t_B - t_A$ then gives $\Delta (t_B - t_A) \approx 0.4$ ps.

After including the temperature variation induced fluctuations, the error of the time offset of clock A and B, which is departed at a fiber link distance of about 10 km, is then $\Delta t_0 \approx 0.54$ ps. Such error determines the synchronization accuracy between the two clocks.

4. Conclusion

In conclusion, we have demonstrated a high-accuracy fiber-based quantum clock synchronization scheme. In this scheme, two distant clocks, which are linked by unknown fiber, can be accurately synchronized with the help of a frequency anti-correlated entangled bichromatic source, quantum correlation measurement setups, and a reference fiber coating, which is placed in the laboratory with its length capable of being precisely controlled and measured. In ideal case, the achievable synchronization accuracy $\Delta t_0$ is only dependent on the bandwidth of the bichromatic spectrum. So the higher the bandwidth of the
biphon spectrum is, the more accurate the synchronization will be. The accuracy is unaffected by the GVD effect or by the fiber propagation length. In reality however, due to the inevitable fiber temperature variations, the accuracy will be limited by the fiber propagation lengths. Based on our scheme, utilizing a biphon source with a bandwidth of 13 nm to synchronize the two clocks at a fiber distance of 10 km, 0.54 ps accuracy can be achieved, which provides a highly promising application in comparing two atomic clocks within a distance of a few to tens of kilometers with an accuracy below a picosecond.

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