Probing high-density behavior of symmetry energy from pion emission in heavy-ion collisions

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ABSTRACT

Within the framework of the improved isospin dependent quantum molecular dynamics (ImIQMD) model, the emission of pion in heavy-ion collisions in the region 1 A GeV as a probe of nuclear symmetry energy at supra-saturation densities is investigated systematically, in which the pion is considered to be mainly produced by the decay of resonances \( \Delta(1232) \) and \( N^*(1440) \). The total pion multiplicities and the \( \pi^-/\pi^+ \), \( \Sigma^-/\Sigma^+ \) and \( K^0/K^+ \) yields are calculated for selected Skyrme parameters SKP, SLy6, Ska and SIII, and also for the cases of different stiffness of symmetry energy with the parameter \( S_{\text{sym}} \). Preliminary results compared with the measured data by the FOPI Collaboration favor a hard symmetry energy of the potential term proportional to \( (\rho/\rho_0)^{\gamma_1} \) with \( \gamma_1 = 2 \).

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Heavy-ion collisions induced by radioactive beam at intermediate energies play a significant role to extract the information of nuclear equation of state (EoS) of isospin asymmetric nuclear matter under extreme conditions. Besides nucleonic observables such as rapidity distribution and flow of free nucleons and light clusters (such as deuteron, triton and alpha, etc.), also mesons emitted from the reaction zone can be probes of the hot and dense nuclear matter. The energy per nucleon in the isospin asymmetric nuclear matter is usually expressed as:

\[
E_{\text{sym}}(\rho, \delta) = E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4)
\]

in terms of baryon density \( \rho = n_\text{n} + n_\text{p} \), relative neutron excess \( \delta = (n_\text{n} - n_\text{p})/(n_\text{n} + n_\text{p}) \), energy per nucleon in a symmetric nuclear matter \( E(\rho, \delta = 0) \) and bulk nuclear symmetry energy \( E_{\text{sym}} = \frac{1}{2} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \bigg|_{\delta=0} \). In general, two different forms have been predicted by some microscopical or phenomenological many-body approaches. One is the symmetry energy increases monotonically with density, and the other is the symmetry energy increases initially up to a supra-saturation density and then decreases at higher densities. Based on recent analysis of experimental data associated with transport models, a symmetry energy of the form \( E_{\text{sym}}(\rho) \approx 31.8(\rho/\rho_0)^{1.95} \) MeV with \( \gamma = 0.69 - 1.05 \) was extracted for densities between 0.1\( \rho_0 \) and 1.2\( \rho_0 \) [1,2]. The symmetry energy at supra-saturation densities can be investigated by analyzing isospin sensitive observables in theoretically, such as the neutron/proton ratio of emitted nucleons, \( \pi^-/\pi^+ \), \( \Sigma^-/\Sigma^+ \) and \( K^0/K^+ \) [2]. Recently, a very soft symmetry energy at supra-saturation densities was pointed out by fitting the FOPI data [3] using IBUU04 model [4].

With the establishment of high-energy radioactive beam facilities in the world, such as the CSR (IMP in Lanzhou, China), FAIR (GSI in Darmstadt, Germany), RIKEN (Japan), SPIRAL2 (GANIL in Caen, France) and FRIB (MSU, USA) [2], the high-density behavior of the symmetry energy can be studied more detail experimentally in the near future. The emission of pion in heavy-ion collisions in the region 1 A GeV is especially sensitive as a probe of symmetry energy at supra-saturation densities. Further investigations of the pion emissions in the 1 A GeV region are still necessary by improving transport models or developing some new approaches.

The ImIQMD model has been successfully applied to treat heavy-ion fusion reactions near Coulomb barrier [5–7]. Recently, Zhang et al. analyzed the neutron–proton spectral double ratios to extract the symmetry energy per nucleon at sub-saturation density with a similar model [8]. To investigate the pion emission, we further include the inelastic channels in nucleon–nucleon collisions.

In the ImIQMD model, the time evolutions of the baryons and pions in the system under the self-consistently generated mean-field are governed by Hamilton’s equations of motion, which read as:

\[
\dot{\mathbf{p}}_i = -\frac{\partial H}{\partial \mathbf{r}_i}, \quad \dot{\mathbf{r}}_i = \frac{\partial H}{\partial \mathbf{p}_i}.
\]

Here we omit the shell correction part in the Hamiltonian \( H \) as described in Ref. [6]. The Hamiltonian of baryons consists of the relativistic energy, the effective interaction potential and the momentum dependent part as follows:
interaction and the local interaction

The effective interaction potential is composed of the Coulomb interaction and the local interaction

\[ U_{\text{int}} = U_{\text{Coul}} + U_{\text{loc}}. \]

The Coulomb interaction potential is written as

\[ U_{\text{Coul}} = \frac{1}{2} \sum_{i,j \neq i} e_i e_j \ \text{erf}(r_{ij}/\sqrt{4L}) \]

where the \( e_i \) is the charged number including protons and charged nucleons. The \( r_{ij} = |\mathbf{r}_i - \mathbf{r}_j| \) is the relative distance of two charged particles.

The local interaction potential is derived directly from the Skyrme energy-density functional and expressed as

\[ U_{\text{loc}} = \int V_{\text{loc}}(\rho(\mathbf{r})) \ d\mathbf{r}. \]

The local potential energy-density functional reads

\[
V_{\text{loc}}(\rho) = \frac{\alpha}{2} \rho^2 + \frac{\beta}{1+\gamma} \rho^{1+\gamma} + \frac{g_{\text{sur}}}{2\rho_0} (\nabla \rho)^2
\]

\[
+ \frac{g_{\text{iso}}}{2\rho_0} \left( \nabla (\rho_n - \rho_p) \right)^2
\]

\[
+ \frac{g_{\text{sym}}}{2\rho_0} \left( \rho_{\text{sym}} \rho^2 \right)
\]

\[
+ \left( \alpha_{\text{sym}} \rho^2 \rho_{\text{sym}} + b_{\text{sym}} \rho^{1+\gamma} \rho_{\text{sym}}^{1+\gamma} + c_{\text{sym}} \rho^{5/3} \rho_{\text{sym}}^{5/3} \right) \delta^2
\]

\[ + g_{\Delta} \rho^{5/3} / \rho_0^{5/3}. \]

where the \( \rho_n, \rho_p \) and \( \rho = \rho_n + \rho_p \) are the neutron, proton and total densities, respectively, and the \( \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) \) is the isospin asymmetry. The coefficients \( \alpha, \beta, \gamma, g_{\text{sur}}, g_{\text{iso}}, g_{\text{sym}}, g_{\Delta} \) are related to the Skyrme parameters \( t_0, t_1, t_2, t_3 \) and \( x_0, x_1, x_2, x_3 \) [6].

The parameters of the potential part in the symmetry energy term are also derived directly from Skyrme energy-density parameters as

\[ \alpha_{\text{sym}} = -\frac{1}{8} (2x_0 + 1) t_0 \rho_0, \]

\[ b_{\text{sym}} = -\frac{1}{48} (2x_1 + 1) t_3 \rho_0^\gamma, \]

\[ c_{\text{sym}} = -\frac{1}{24} \left( \frac{3}{2} \pi^2 \right)^{2/3} \rho_0^{5/3} \left[ 3t_1 x_1 - t_2 (5x_2 + 4) \right]. \]

The momentum dependent term in the Hamiltonian is the same of the form in Ref. [9] and expressed as

\[ U_{\text{mom}} = \frac{\delta}{2} \sum_{i,j \neq i} \frac{\rho_{ij}}{\rho_0} \left[ \ln(\epsilon (\mathbf{p}_i - \mathbf{p}_j)^2 + 1) \right]^2. \]

with

\[ \rho_{ij} = \frac{1}{(4\pi L)^{3/2}} \exp \left[ -\frac{(|\mathbf{r}_i - \mathbf{r}_j|^2)}{4L} \right]. \]

which does not distinguish between protons and neutrons. Here the \( L \) denotes the square of the pocket wave length, which is dependent on the mass number of the nucleus. The parameters \( \delta \) and \( \epsilon \) were determined by fitting the real part of the proton–nucleus optical potential as a function of incident energy.

In Table 1 we list the ImIQMD parameters related to several typical Skyrme forces after including the momentum dependent interaction. The parameters \( \alpha, \beta, \gamma \) are re-determined in order to reproduce the binding energy \( (E_B = -16 \text{ MeV}) \) of symmetric nuclear matter at saturation density \( \rho_0 \) and to satisfy the relation \( \frac{\partial E}{\partial \rho} \mid_{\rho=\rho_0} = 0 \) for a given incompressibility. Combined Eq. (7) with the kinetic energy part, the symmetry energy per nucleon in the ImIQMD model is given by

\[ E_{\text{sym}}(\rho) = \frac{1}{3} \frac{h^2}{2m} \left( \frac{3}{2} \pi^2 \rho \right)^{2/3} + a_{\text{sym}} \frac{\rho}{\rho_0} \gamma
\]

\[ + b_{\text{sym}} \left( \frac{\rho}{\rho_0} \right)^\gamma + c_{\text{sym}} \left( \frac{\rho}{\rho_0} \right)^{5/3}. \]

More clearly compared with other transport models, the symmetry energy can be expressed as

\[ E_{\text{sym}}(\rho) = \frac{1}{3} \frac{h^2}{2m} \left( \frac{3}{2} \pi^2 \rho \right)^{2/3} + \frac{1}{2} C_{\text{sym}} \left( \frac{\rho}{\rho_0} \right)^{\gamma}. \]

The value \( \gamma_s = 1 \) is used in IQMD model [10,11]. In Fig. 1 we show a comparison of the energy per nucleon in symmetric nuclear matter with and without the momentum dependent potentials in the left panel and the nuclear symmetry energy in the right panel for different cases of Skyrme forces SkP, SkY, Ska and SIII from Eq. (10), \( \gamma_s = 0.5 \) (soft) and 2 (hard) with \( C_{\text{sym}} = 32 \text{ MeV} \) in Eq. (11), and also compared with the form \( E_{\text{sym}} = 31.6(\rho/\rho_0)^{3/2} \text{ MeV} \) \( (\mu = 0.5 \text{ and } \mu = 2) \) [11].

Analogously to baryons, the Hamiltonian of pions is represented as

\[ H_{\pi} = \sum_{i=1}^{N_{\pi}} \left( \mathbf{p}_i^2 + m^2_{\pi} + V_{\text{Coul}}(i) \right), \]

where the \( \mathbf{p}_i \) and \( m_{\pi} \) represent the momentum and the mass of the pions. The Coulomb interaction is given by
\[ V_{ij}^{\text{Coul}} = \sum_{j=1}^{N_{B}} E_{Fj}^{\text{Coul}} \]  

where the \( N_{p} \) and \( N_{B} \) is the total number of pions and baryons including charged resonances. Thus, the pion propagation in the whole stage is guided essentially by the Coulomb force. The in-medium pion potential in the mean field is not considered in the model. However, the inclusion of the pion optical potential based on the perturbation expansion of the \( \Delta \)-hole model gives negligible influence on the transverse momentum distribution [12].

The pion is created by the decay of the resonances \( \Delta(1232) \) and \( N^{*}(1440) \) which are produced in inelastic NN scattering. The cross section of direct pion production is very small in the considered energies and not included in the model [13]. The reaction channels are given as follows:

\[
NN \leftrightarrow N\Delta, \quad NN \leftrightarrow NN^*, \quad NN \leftrightarrow \Delta\Delta, \\
\Delta \leftrightarrow N\pi, \quad N^* \leftrightarrow N\pi. \tag{14}
\]

The cross sections of each channel to produce resonances are parameterized by fitting the data calculated with the one-boson exchange model [14]. In the 1 A GeV region, there are mostly \( \Delta \) resonances which disintegrate into a \( \pi \) and a nucleon, however, the \( N^{*} \) yet gives considerable contribution to the high energetic pion yield. The energy and momentum dependent decay width is used in the calculation [15].

Pion meson in heavy-ion collisions is mainly produced at supersaturation densities of compressed nuclear matter larger than the normal density \( \rho_{0} \). The production of pions is influenced by the \( \Delta(1232) \) and the Fermi motion of baryons in the vicinity of the threshold energies. The \( \pi^{-}/\pi^{+} \) ratio is a sensitive probe to extract the high-density behavior of the symmetry energy per energy. Shown in Fig. 2 is a comparison of the measured total pion multiplicity and \( \pi^{-}/\pi^{+} \) yields by the FOPI Collaboration in central \( ^{197}\text{Au} + ^{197}\text{Au} \) collisions [3] and the results calculated by IQMD model [10] as well as by the ImIQMD model for Skyrme parameters SkP, SkY6, Ska and SII, which correspond to different modulus of incompressibility as listed in Table 1. The total multiplicity of pion is mainly determined by the cross sections of the channels \( NN \leftrightarrow N\Delta \). The ImIQMD model with four Skyrme parameters predicts rather well the total yields at higher incident energies, but slightly overestimates the values near threshold energies, which may be influenced by the in-medium cross sections. In this work, we use the in-vacuum cross sections of nucleon–nucleon elastic and inelastic collisions. Reasonable consideration of the in-medium inelastic collisions in producing \( \Delta \) and \( N^{*} \) is still an open problem in transport models, which have been performed in Giessen-BUU model [16]. Using the isobar model, one gets the ratio \( \pi^{-}/\pi^{+} = 1.95 \) for pions from the \( \Delta \) resonance, and \( \pi^{-}/\pi^{+} = 1.7 \) from the \( N^{*} \) for the system \( ^{197}\text{Au} + ^{197}\text{Au} \) [17]. These relations are globally valid, i.e. independent of the pion energy. On the other hand, the statistical model predicts that the \( \pi^{-}/\pi^{+} \) ratio is sensitive to the difference in the chemical potentials of neutrons and protons by the relation \( \pi^{-}/\pi^{+} \propto \exp(2(\mu_{n} - \mu_{p})/T) = \exp(84E_{\text{sym}}(\rho)/T) \), where the \( T \) is nuclear temperature [18]. The observed energy dependence of the \( \pi^{-}/\pi^{+} \) ratio is due to the re-scattering and absorption process of pions and nucleons in the mean field of the compressed nuclear matter. We use the free absorption cross sections in collisions of pions and nucleons by fitting the experimental data. The branch ratio of the charged \( \pi \) and \( \pi^{\pm} \) is determined by the Clebsch–Gordan coefficients with the decay of the resonances \( \Delta(1232) \) and \( N^{*}(1440) \). The \( \pi^{-}/\pi^{+} \) ratio is sensitive to the stiffness of the symmetry energy at the lower incident energies. The ImIQMD model can predict the decrease trend of the \( \pi^{-}/\pi^{+} \) ratio with incident energy. While the ImIQMD model with different Skyrme parameters gives the same excitation functions of the total pion multiplicity owing to the same cross sections in the production of pions and resonances for each case, the \( \pi^{-}/\pi^{+} \) yields is different resulting from the symmetry energy.

The compressed nuclear matter with central density about two times of the normal density is formed in heavy-ion collisions in the 1 A GeV region. To extract more information of symmetry energy in heavy-ion collisions from the pion production, in Fig. 3 we calculated the time evolution of average central density from low to high incident energies and the excitation functions of the total pion\( \gamma \) ratio with the force SkY6, but different stiffness of the symmetry energy which corresponds to hard (\( \gamma_{s} = 2 \)), linear (\( \gamma_{s} = 1 \)), soft (\( \gamma_{s} = 0.5 \)) and softer (SII), and also compared with IQMD results [10] as well as the FOPI data [3]. The ImIQMD model gives larger values of \( \pi^{-}/\pi^{+} \) than the ones calculated by IQMD, which mainly results from the cross section of the channel \( N\gamma \rightarrow \Delta \) and the larger coefficient \( C_{\text{sym}} \). We considered the pion absorption process according to the Br"{u}t–Wigner formula with the cross section given in Ref. [13]. Our calculations show that a stiff symmetry energy is close to experimental data. The results does not support a very soft symmetry energy at high-density from analyzing the same experimental data reported in Ref. [4]. Situation is different in IBUU04 model, each nucleon in the evolution is
enforced by the symmetry potential associated with isospin and momentum. Inversely, a transport model reported in Ref. [19] also predicted the larger ratios for stiffer symmetry energy from the analysis of the $\pi^-/\pi^+$ and $K^0/K^+$ yields. The influence of the symmetry energy on pion production in heavy-ion collisions is also studied from the distribution of transverse momentum of the total charged pions and the ratio $\pi^-/\pi^+$ for the cases of stiff and soft symmetry energies as shown in Fig. 4. The $\pi^-$ mesons are mostly produced from neutron–neutron collisions, and for a stiff symmetry energy, a wider high-density zone is formed in the calculation of the ImIQMD model. The larger $\pi^-/\pi^+$ ratio is also clear in the momentum distribution and the larger errors at the
The decrease of the $N/Z$ ratio with different stiffness of the symmetry energy is shown in Fig. 5 as a function of $N/Z$ in the reaction systems for head on collisions at incident energy $E_{lab} = 0.4$ A GeV and 1.5 A GeV, respectively. The FOPI data [3] and the results calculated by IQMD model [10] are also given for a comparison. Experimental data show that an increase trend of the $\gamma_1 = 2$ is close to the experimental data. The phenomena can be explained from the fact that the symmetry energy enhancement the $N/Z$ ratio in the high-density region at lower incident energy. The decrease of the $\gamma_1$ ratio with the incident energy is mainly owing to the production of pions from secondary nucleon–nucleon collisions, such as a neutron converts a proton by producing $\pi^-$. Subsequent collisions of the energetic proton can convert it back to neutron by producing $\pi^+$. One can see that the stiffer symmetry energy is also close to the experimental data. Recently, a moderately soft symmetry energy with $\gamma_1 \simeq 0.9 \pm 0.3$ was extracted from the analysis of neutron–proton elliptic flow of the FOPI/LAND data for the reaction $^{197}$Au + $^{197}$Au using the UrQMD model [20]. Further experimental works associated transport models should be performed in more details to get reliable information of the high-density trend of the symmetry energy in heavy-ion collisions.

In summary, the pion production in heavy-ion collisions in the region 1 A GeV is investigated systematically by using the ImIQMD model. The total multiplicity of produced pion and the $\pi^-/\pi^+$ ratio in central collisions are calculated for the selected Skyrme parameters SkP, Sly6, SkI, SkII which correspond to different moduli of incompressibility of symmetric nuclear matter and different cases of the stiffness of symmetry energy, and compared them with the experimental data by the FOPI collaborations as well as IQMD results. The $\pi^-/\pi^+$ excitation functions for the reaction $^{197}$Au + $^{197}$Au and the dependence of the $\pi^-/\pi^+$ ratio on $N/Z$ of reaction systems at energy 0.4 A GeV are compared with the force Sly6, but different stiffness of the symmetry energy. Calculations show that a stiffer symmetry energy of the potential term with $\gamma_1 = 2$ is close to the experimental data.

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