Energy-aware utility optimisation for joint multi-path routing and MAC layer retransmission control in TDMA-based wireless sensor networks

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Abstract: This paper formulates a novel energy-aware utility optimisation (EUO) problem for joint multi-path routing and medium access control (MAC) layer retransmission control in time division multiple access (TDMA)-based wireless sensor networks (WSNs). As the EUO problem is non-convex and inseparable, we propose a heuristic method called Successive Alternating Convex Approximation (SACA) to approximately solve the problem. More precisely, we decompose the EUO problem into two alternating convex sub-problems, namely energy-aware multi-path routing and retransmission control. On one hand, given the time slot allocation, the multi-path routing balances the energy consumption within WSNs. On the other hand, provided the traffic distribution, retransmission control enhances the data reliability in the most energy-efficient way. These two sub-problems are solved recursively by means of dual decomposition. The sequence of optimal solutions to these two sub-problems is shown to converge to some steady-state point that approximates a solution to the EUO problem.

Keywords: wireless sensor networks; multi-path routing; retransmission control; convex optimisation.

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1 Introduction

Wireless Sensor Networks (WSNs) are characterised by multi-hop lossy wireless links and severely resource constrained nodes. Among the resource constraints, energy is probably the most crucial one since sensor nodes are typically battery powered and the lifetime of the battery imposes a limitation on the operation hours of wireless sensor networks (Akyildiz et al., 2002). Thus a lot of works concentrate on the design of energy-aware protocols in order to prolong the operational time of WSNs (Chang and Tassiulas, 2004; Hou et al., 2008; Madan and Lall, 2006). Besides energy constraints, transmission reliability is another concern in data-centric WSNs (Willig et al., 2008; Deb et al., 2003; Zhu et al., 2007; Zheng et al., 2011; Ovalle-Martinez et al., 2006; Korad and Sivalingam, 2006; Tetzcan and Wang, 2007; AboElFotoh et al., 2007; Pang et al., 2008; Chen et al., 2008; Chen et al., 2009; Bein et al., 2009), such as industrial wireless networks (Willig et al., 2008; HARTCOMM, 2007; Liang et al., 2011). To improve the performance of WSNs, data delivery in WSNs should be both energy efficient and reliable.

In this paper, we study multi-path routing and MAC layer retransmission control to provide energy efficient and reliable data delivery for TDMA-based WSNs. The motivation of this paper comes from the following two intuitive reasons. First, load-balancing may be the most effective method in current energy-aware routing algorithms to prevent the high energy consumption of some bottleneck nodes in WSNs. Multi-path routing schemes are natively appropriate for splitting the source rates among different routes for load balancing. Second, for reliable data delivery in TDMA-based WSNs, MAC layer (link level) retransmission is usually adopted. More precisely, the transmitter node receives an Acknowledgement (ACK) if its data packet is correctly received by its destined receiver. Otherwise, retransmissions happen in the following time slots until the reception of an ACK by the transmitter or the maximum number of retransmissions being met, which was first named as the replicator factor in Zheng et al. (2011). Effective MAC layer retransmission control could guarantee high end-to-end reliable transmissions while prolonging the network lifetime (Zheng et al., 2011). However, this work, the joint optimisation over the space of routes and replicator factors, is nontrivial. The difficulty is confirmed by the fact that routing variables and retransmission variables are tightly coupled in the network wide, which results in the joint optimisation problem non-convex.

This paper proposes a heuristic approach to approximately solve the non-convex joint multi-path routing and retransmission control problem. We assume that the replicator factors are fixed so that the problem reduces to a convex routing sub-problem. The optimal solution to this sub-problem is then fed to the other convex retransmission control sub-problem where flow rates on routes are assumed to be fixed. This alternating procedure is repeated and the resulting sequence of optimal solutions to the convex sub-problems is shown to converge to a steady-state point that approximates the optimal solution to the EUO problem. This solving procedure is the so-called SACA. The implementation of SACA is especially appropriate for industrial automation-oriented WSNs. A brief illustration on the implementation of SACA over WirelessHART-based WSNs is also given.

Our contribution is therefore threefold. First, for TDMA-based WSNs, we formulate a novel EUO problem for joint multi-path routing and retransmission control. Second, an effective heuristic approach, SACA, is proposed to approximately solve the EUO problem, which is non-convex and inseparable. Third, the EUO in this paper is recommended for industrial wireless networks, such as WirelessHART-based WSNs.
2 Related work

This paper could be considered as a substantial extension of work of Zheng et al. (2011). For TDMA-based WSNs, usually the maximum number of retransmissions is configured by heuristic ways. Heuristic ways are generally inefficient, since large values may lead into a huge waste of energy and small values may not guarantee the desired transmission reliability. Zheng et al. (2011) proposed a reliability-Lifetime trade-off framework to study MAC layer retransmission control in a mathematical manner. This paper combines multi-path routing and MAC layer retransmission control in Zheng et al. (2011) to provide energy efficient and reliable data delivery for TDMA-based WSNs. There is also research on the network utility maximisation (NUM) for both wired (Kelly et al., 1998; Low and Lapsley, 1999; Shen and O’Hare, 2007), and wireless networks (Chiang et al., 2007; Lin et al., 2006). NUM framework that successfully addressed the rate control problem in wired networks was first proposed in the seminal paper by Kelly et al. (1998). Chiang et al. (2007) proposed the Layering As Optimisation Decomposition (LAOD) framework for wireless networks. Nama et al. (2006) extended the traditional energy-aware research and jointly studied utility maximisation and energy conservation problem for WSNs. After that, many scholars devoted to the study on the rate-lifetime trade-off framework in WSNs (Zhu et al., 2008; Chen et al., 2010; Zheng et al., 2010a; Zheng et al., 2010b). Most of them concentrated on the development of fully distributed implementation or the development of extended models (Nama et al., 2006). This work is similar to the rate-lifetime trade-off optimisation in WSNs (Nama et al., 2006; Zhu et al., 2008; Chen et al., 2010; Zheng et al., 2010a; Zheng et al., 2010b). However, the modeling method and the context of this work are totally different. The objective (i.e., EUO) and optimising variables (i.e., flow rates and replicator factors) in this paper are different from those (i.e., rate-lifetime trade-off and power, scheduling, source rates and so on) of (Nama et al., 2006; Zhu et al., 2008; Chen et al., 2010; Zheng et al., 2010a; Zheng et al., 2010b).

3 Modelling and problem statement

3.1 Network model

Consider a WSN with an established network topology, in which all links share a common channel with a fixed error rate $p$. We represent the WSN as a directed acyclic graph $G(V, L, R)$, where $V$ denotes the set of sensor nodes, $L$ denotes the set of logical links, and $R$ denotes the set of routes. Let $L_{out}(v)$ denote the set of outgoing links from node $v$, $L_{in}(v)$ the set of incoming links to node $v$, $L(l)$ the set of links that route $r$ passes through, and $R(l)$ the set of routes whose data flows through link $l$. In this paper, a TDMA-based MAC is employed. Time is divided into periodic MAC superframes, and each MAC superframe consists of multiple time slots. Each sensor is allowed to transmit in its allocated time slots in each MAC superframe so that no collision could occur. Thus the transmission failure could only be due to channel errors whose negative effect on transmission reliability could be alleviated by retransmission control. Source nodes periodically transmit sensed data to the sink node through multiple paths and also relay other nodes’ data according to predefined routes. How the sink node schedules the transmission of links is out of the scope of this paper, since TDMA scheduling in wireless networks is an important topic in its own right. Hence we assume that all the sensed data could arrive at the sink within one superframe for simplicity.

In our network model, proactive routing is preferred since the network topology of the WSN is static and the source traffic is periodic (fixed). Let $R(v) \subseteq R$ be the set of all routes for sensor node $v$, $v \in V$. Denoting the source rate of sensor node $v$ by $s_v$, we have the following flow conservation constraint:

$$s_v = \sum_{r \in R(v)} y_r, \forall v \in V$$  \hspace{1cm} (1)

where $y_r$ denotes the flow rate on route $r$.

Let $\varepsilon_v$ and $\varepsilon_r$ denote the energy consumed per bit in hardware in sensing and receiving data, respectively. We assume that all nodes have identical power dissipation characteristics in sensing and receiving. Let $\varepsilon_r'$ denote the energy consumed per bit in transmitting on link $l$. $\varepsilon_r'$ is given by $\varepsilon_r' = \mu + \eta d_l^N$, where $\mu$ is the energy cost of transmit electronics of node $v$, $\eta$ is a coefficient term corresponding to the energy cost of transmit amplifier, and $d_l$ is the distance between two terminal nodes of link $l$. $N$ is the path loss factor, $2 \leq N \leq 4$. Then the total average power dissipated in the node $v$ is given by

$$a_v = \sum_{l \in L_{out}(v)} \varepsilon_r' f_r \alpha_l + \sum_{l \in L_{in}(v)} \varepsilon_r f_r \alpha_l + \varepsilon_v s_v, \forall v \in V$$  \hspace{1cm} (2)

where $f_r := \sum_{r \in R(l)} y_r$ denotes the average flow rate on link $l$.

Let $P_r$ denote the end-to-end transmission reliability of route $r$. $P_r$ is calculated as the product of successful transmission probabilities of all the links on route $r$, i.e.,

$$P_r = \prod_{l \in L(r)} (1 - p^N), r \in R$$  \hspace{1cm} (3)

where $\alpha_l$ denotes the replicator factor of link $l$. With (3) in hand, the correctly received data rate $c_r$ on route $r$ can be represented as follows,

$$c_r = y_r P_r, r \in R.$$  \hspace{1cm} (4)
For concise exposition, we denote vectors \((y_i, \, (\alpha_i), \, (P_i), \, (c_i), \, and \, (a_i))\) by \(y, \, \alpha, \, P, \, c, \, and \, a\), respectively. In addition, there may be some regulatory limitations that impose individual constraints on replicator factors and flow rates. These constraints will be captured by maximising our objective function over the following set \(B := \{(y, \alpha, \, P, \, c, \, a) : y^\mu \leq y, \, y^w \leq y^w, \alpha^u \leq \alpha^u, \, P^u \leq P^u, \, c^w \leq c^w, \, a^w \leq a^w, \, r \in R, \, l \in L, \, v \in V\}\).

3.2 Energy-aware utility optimisation

In this sub-section, we propose the EUO problem by incorporating the transmission reliability and a suitably chosen energy cost into the traditional NUM problem (Kelly et al., 1998),

\[
U = \max_{(y,\alpha, P, c, a) \in B} U(c,a) \quad \text{s.t.} \quad (1),(2),(3),\text{and} \, (4) \tag{5}
\]

where \(U(c,a)\) is a utility-cost function defined to be

\[
U(c,a):=\sum_{v \in V} \omega_v U_v(c_v) - \sum_{v \in V} \lambda_v W_v(a_v). \tag{6}
\]

The utility function \(U_v(x)\) and the penalty function \(W_v(x)\) are assumed both differentiable and increasing, but strictly concave and strictly convex, respectively. The weight vector \(\omega=(\omega_1, \ldots, \omega_v)\) is any fixed positive vector. The variable \(\lambda_v = \lambda/E_v, \, v \in V\), is the power price of node \(v\), where \(\lambda > 0\) is a given system constant and \(E_v > 0\) is the energy available at node \(v\). The cost of node \(v\) is then equal to the product of the power price \(\lambda_v\) and its power penalty.

In the following, we use the following family of utility functions, parameterised by \(\beta \geq 1\) (Mo Walrand, 2000):

\[
U^\beta(x) = -W^\beta\left(\frac{1}{x}\right) = \begin{cases} 
(1-\beta)x^{1-\beta}, & \text{if } \beta > 1 \\
\log x, & \text{if } \beta = 1
\end{cases} \tag{6}
\]

Utility function (6) is widely used for the resource allocation in wired and wireless networks, since it provides a useful tool for resource allocation by controlling the trade-off between efficiency and \(\beta\)-proportional fairness\(^1\) through parameter \(\beta\).

4 Successive alternating convex approximation

Obviously, because of constraints (2) and (4), the feasible domain of problem (5) is non-convex. Thus, in general it is extremely difficult to obtain the optimal solution to problem (5). In the following, we propose SACA that decomposes problem (5) into two convex sub-problems (energy-aware multi-path routing and energy-aware retransmission control) and solve these two sub-problems recursively until some convergence criterion is satisfied.

4.1 Energy-aware multi-path routing

Assuming the replicator factors \(\alpha_v, \, l \in L\), are fixed, the problem (5) reduces to a convex sub-problem. In order to show this, we rewrite it as

\[
\tilde{U}(\alpha) := \max_{y \in Y} \sum_{r \in B} \tilde{\omega}_r U_r(y_r) - \sum_{r \in B} \tilde{\lambda}_r W_r(a_r(y)) \tag{7}
\]

\[
s.t. \quad s_y = \sum_{r \in B_{r\alpha}} y_r, \, v \in V \tag{8}
\]

where \(B_r\) denotes the projection of \(B\) on \(y\), \(\tilde{\omega}_r\) and \(a_r(y)\) are defined as follows:

\[
\tilde{\omega}_r := \omega_r U_r(P_r)(1-\beta) \tag{9}
\]

\[
a_r := \frac{\tilde{\omega}_r U_r(y_r) - \sum_{l \in L} \sum_{r \in B_{r \alpha}} \xi \alpha_l y_l + \sum_{l \in L} \xi l \alpha_l y_l + \xi s_y}{s_y - \sum_{r \in B_{r \alpha}} y_r} \tag{10}
\]

Obviously, \(a_r(y)\) is an affine function of \(y\), and thus the composition function \(-W^\beta(a_r(y))\) is concave with respect to \(y\). Then the problem (7) is strictly convex problem and the dual decomposition method could be used to solve it with zero dual gap.

We introduce dual variable \(\xi = (\xi_v), \, v \in V\), for the constraint (8) and construct the partial Lagrangian function

\[
L(\xi, y) = \sum_{r \in R} \tilde{\omega}_r U_r(y_r) - \sum_{r \in R} \tilde{\lambda}_r W_r(a_r(y)) - \sum_{v \in V} \xi_v (s_v - \sum_{r \in B_{r \alpha}} y_r). \tag{11}
\]

Then, the dual function of problem (7) is

\[
D(\xi) = \max_{y \in Y} L(\xi, y) \tag{12}
\]

and the dual problem is

\[
\min_{\xi} D(\xi). \tag{13}
\]

To solve the dual problem (13), first we have to solve the dual function (12). For given \(\xi\), \(L(\xi, y)\) is differentiable with respect to \(y\), and the component of its gradient \(G_v(y)\) is shown as follows:

\[
G_v(y) = \frac{\partial L(\xi, y)}{\partial y_v} = \frac{\partial U_v(y_v)}{\partial y_v} - \sum_{r \in B_{r \alpha}} \tilde{\lambda}_r W_r^\beta(a_r(y_v)) + \xi \tag{14}
\]

where \(V(r)\) denotes the set of sensors that route \(r\) passes through and \(v(r)\) denotes the source node of route \(r\). At the \(t+1\)th iteration, the formula for updating \(y_v\) can be stated as

\[
y_v(t+1) = y_v(t) + \gamma G_v(y(t)) \tag{15}
\]

where \(\gamma\) is the positive constant stepsize and \(\Pi_{B_r} [\ast] = \max\{y^\text{max}_v, \min\{\ast, y^\text{min}_v\}\} \) .
We are now ready to solve the dual problem (10). The strict convexity of problem (7) implies the differentiability of $D(\xi)$. At the $n+1$th iteration, provided $y(n)$, each source node, say node $v$, $v \in V_r$, updates the dual variable $\xi_v$ as follows:

$$
\xi_v(n+1) = \xi_v(n) + \gamma(s_v - \sum_{v \in N(v)} y_v(n))
$$

where $\gamma$ is the positive stepsize.

In the following, we formally conclude the update procedure (see Algorithm 1).

**Algorithm 1: Energy-aware Multi-path Routing**

**Require:** $\alpha \in \mathbb{R}$, $P$, $\delta > 0$, $\lambda > 0$, $\gamma > 0$, $t=1$, $n=1$, $\forall r \in R$, $v \in V_r$.

1: repeat
2: repeat
3: Each sensor node $v$ computes $\lambda_v W_v(a_v(y)) \frac{\partial a_v(y)}{\partial y_v}$ and piggy-backs this information on the data.
4: Sink node collects the gradient information
   $\sum_{v \in V_r} \lambda_v W_v(a_v(y)) \frac{\partial a_v(y)}{\partial y_v}$ from the received packets for each route $r$ and broadcasts it.
5: Each sensor $v$ reads the broadcast and updates flow rates of its outgoing routes $y_v(t)$, $r \in R_v$, as (12).
6: $t = t + 1$.
7: until convergence.
8: $y(n) = y(t)$.
9: Each source node $v$ updates dual variable $\xi_v(n)$ according to (13).
10: $n = n + 1$.
11: until convergence.
12: Output $y = y(n)$.

Comment 1: (1) All steps in Algorithm 1 are distributed except Step 4. In Step 4, sink node acts as an information centre that is responsible for collecting the gradient information for all routes. Thus Algorithm 1 is a partially distributed algorithm. (2) Algorithm 1 is based on the gradient projection method, and thus convergence is guaranteed provided the step size $\gamma$ is sufficiently small (Bertsekas, 1999).

4.2 Energy-aware retransmission control

Assuming the flow rates $y_v$, $r \in R$, are fixed, the problem (5) reduces to the other convex sub-problem shown as follows:

$$
U(y) = \max_{(P, a) \in B_{(P, a)}} \sum_{v \in V_r} B_r U_r(y, a) - \sum_{v \in V_r} \lambda_v W_v(a_\alpha)
$$

s.t. $P - \prod_{v \in V_r} (1 - p_v^{\text{out}}) \leq 0, r \in R$

where $B_{(P, a)}$ denotes the projection of $B$ on $(P, a)$, $\bar{\theta}_r$ and $a_\alpha$ are defined as follows:

$$
\bar{\theta}_r := \omega U_r(y_v(1 + \beta))
$$

$$
a_\alpha := \sum_{l \in L(v)} e_{l, \alpha} + e_{l, \alpha} + e_{s, \alpha}.
$$

Comment 2: In problem (14), we relax the constraint (15), which will not change the optimal solution of problem (14), since the utility function $U_r(x)$ is strictly monotone increasing and consequently the constraint (15) is always tight.

Different from problem (7), problem (14) is non-convex because of the constraint (15). In constraint (15), all replicator factors $a_{l, \alpha}$ ($l\in L(r)$) on route $r$ ($r \in R$) are tightly coupled by the product term. However, in the following problem (14) will be shown essentially convex.

We first take the logarithm of both sides of the constraint in (15) and make a logarithm substitution of variables: $P_r = \log(P_r)$ and $U_r(P_r) = U_r(e^{P_r})$. Then the problem (14) is transformed as follows:

$$
P_r - \prod_{l \in L(v)} (1 - p_l^{\text{out}}) \leq 0, r \in R,
$$

$$
P_r - \sum_{l \in L(v)} \log(1 - p_l^{\text{out}}) \leq 0, r \in R,
$$

where $B_{(P, a)} := \{(y_v, \alpha, P, c, a) ; y_v^{\text{min}} \leq y_v \leq y_v^{\text{max}}, a_{l, \alpha}^{\text{min}} \leq a_{l, \alpha} \leq a_{l, \alpha}^{\text{max}}, \log(\rho_r^{\text{max}}) \leq \log(\rho_r^{\text{out}}), c_v^{\text{min}} \leq c_v \leq c_v^{\text{max}}, \alpha_v^{\text{min}} \leq a_v \leq a_v^{\text{max}}, r \in R, l \in L, v \in V_r \}$ and $P_r = (\bar{P}_r)$.

It is easy to verify that $U_r(\bar{P}_r)$ is a concave function when $U_r(x)$ is in the form (6). Further, we can easily verify that $F_r(\bar{P}_r, \alpha) = P_r - \sum_{l \in L(v)} \log(1 - p_l^{\text{out}})$ is convex with respect to $(\bar{P}_r, \alpha)$. Hence, the dual decomposition method could be used to solve the problem (16), since the dual gap of problem (16) is zero.

We introduce dual variable $\mu = (\mu_v)$, where $\mu_v \geq 0$, $r \in R$, for the constraint (17) and build the partial Lagrangian function

$$
L(\mu, \bar{P}, \alpha) = \sum_{r \in R} B_r U_r(\bar{P}_r) - \sum_{v \in V} \lambda_v W_v(a_{\alpha})
$$

$$
+ \sum_{r \in R} \mu_r (\bar{P}_r - \sum_{l \in L(v)} \log(1 - p_l^{\text{out}}))
$$

$$
+ \sum_{r \in R, l \in L(v)} \mu_r \log(1 - p_l^{\text{out}})
$$

Then, the dual function of problem (16) is

$$
\bar{D}(\mu) = \max_{(P, a) \in B_{(P, a)}} L(\mu, \bar{P}, \alpha)
$$
and the dual problem is
\[
\min_{\mu \geq 0} D(\mu)
\]
where ‘\(\geq\)’ denotes the component-wise inequality. To solve the dual problem (19), first we have to handle the dual function (18). For the given \(\mu, D(\mu)\) can be decomposed into two separate sub-problems
\[
\max_{\Phi} \sum_{l \in L} (\beta_l \mathcal{G}_l(\overline{P}_l) - \mu, \overline{P}_l)
\]
(20)
and
\[
\max_{\alpha} \mathcal{Q}(\alpha)
\]
(21)
where
\[
\mathcal{Q}(\alpha) = \sum_{n \in V} \sum_{l \in L(n)} \mu_l \log(1 - p_n^a) - \sum_{r \in R} \lambda_r P_r(\alpha(\alpha)).
\]
For the sub-problem (20), each source node conducts the maximisation in parallel:
\[
\overline{P}_r = \arg \max_{\overline{P}_r} \sum_{l \in L} (\beta_l \mathcal{G}_l(\overline{P}_l) - \mu, \overline{P}_l).
\]
Due to the non-negativity nature of \(\mu\), the sub-problem (21) is strictly concave with respect to \(\alpha\). Since \(\mathcal{Q}(\alpha)\) is differentiable with respect to \(\alpha\), its gradient \(\overline{G}_l(\alpha)\) exists and its component \(\overline{G}_l(\alpha)\) is shown as follows:
\[
\overline{G}_l(\alpha) = \sum_{n \in V(l)} \mu_n \left( \frac{\log(p_n^a)}{p_n^a - 1} \right) - \sum_{r \in R} \lambda_r P_r(\alpha(\alpha)) \frac{\partial a_r(\alpha)}{\partial a_r}
\]
(23)
where \(V(l)\) denotes the set of terminal sensors of link \(l\). At the \(t+1\)th iteration, the formula for updating \(\alpha_l\) can be stated as
\[
\alpha_l(t+1) = \Pi_{\alpha_l} \left[ \alpha_l(t) + \gamma(\overline{G}_l(\alpha(t))) \right]
\]
(24)
where \(\Pi_{\alpha_l} \{ \} = \max \{ \alpha_l^{\max}, \min \{ \alpha_l^{\min} \} \} \).

We are now ready to solve the dual problem (19). At the \(n+1\)th iteration, provided \((\overline{P}(n), \alpha^*(n))\), each source node, say node \(v \in V\), updates the dual variables \(\mu_r \in \mathcal{R}(v)\) as follows:
\[
\mu_r(n+1) = \left[ \mu_r(n) + \delta(n)(\overline{P}_r(n) - \sum_{l \in L_r} \log(1 - p_n^a(n))) \right] + \mu_r(n)
\]
(25)
where \(\delta(n)\) is the positive stepsize in the \(n+1\)th iteration and \(\bullet^+ = \max \{ \bullet, 0 \} \).

In the following, we summarise the above procedure and formally propose Algorithm 2.

**Algorithm 2: Energy-aware retransmission control**

**Require:** \(y, \overline{P}_r, \lambda, \alpha > 0, \ t = 1, n = 1, \sigma > 0, \forall r \in \mathcal{R}, v \in V\).

1: repeat
2: Each source node \(v\) computes \(\overline{P}_r(n), r \in \mathcal{R}(v)\) according to (22).
3: repeat
4: Each sensor node \(v\) computes \(\lambda_r P_r(\alpha(\alpha)) \frac{\partial a_r(\alpha)}{\partial a_r}\) and piggy-backs this information on ACK to its upstream neighbours.
5: Each sensor \(v\) computes \(G_i(\alpha(t)), l \in L_{out}(v)\) as (23) and updates replicator factors of its outgoing links as (24).
6: \(t = t + 1\).
7: until convergence.
8: \(\alpha^*(n) = \alpha(t)\).
9: Sink node calculates \(\sum_{l \in L(v)} \log(1 - p_n^a(\alpha))\) for each route \(r, r \in \mathcal{R}\) and broadcasts it. Each sensor \(v\) reads the broadcast and computes dual variables \(\mu_r(n), r \in \mathcal{R}(v)\) as (25). The resulting \(\mu_r(n), r \in \mathcal{R}(v)\) are then piggy-backed on the source data on route \(r\).
10: \(n = n + 1\).
11: until convergence.
12: Output \((\overline{P}(\overline{\alpha}), \alpha^*(n), \mu(n))\).

The convergence and optimality of Algorithm 2 can be established by the following Theorem 1.

**Theorem 1:** Algorithm 2 converges to an optimal dual variable \(\mu^*\) that solves dual problem (19). Furthermore, at \(\mu^*\), solutions to problems (20) and (21), \(\overline{P}\) and \(\alpha^*\), together form the optimal solution to problem (16), namely, \((\overline{P}, \alpha^*)\) is the optimal solution to problem (14).

**Proof:** By Danskin’s theorem (Bertsekas, 1999),
\[
\frac{\partial E(\mu, \overline{P}, \alpha)}{\partial \mu} = \sum_{l \in L} \log(1 - p_n^a) - \overline{P}_r.
\]
Hence, (25) is a subgradient algorithm for the dual problem (19) as derived in the steps preceding the description of Algorithm 2. There thus exists a step size \(\delta(n)\) (e.g., \(\delta(n) = 1/n\)) that guarantees \(\mu(n)\) converges to an optimal dual solution (Bertsekas, 1999). Problem (16) is a convex optimisation problem and each of problems (20) and (21) has unique solution at \(\mu^*\). Hence, \((\overline{P}, \alpha^*)\) is the optimal solution to problem (16). This completes the proof.

**Comment 3:** (1) Similar to Algorithm 1, Algorithm 2 is also partially distributed because of Step 9. (2) Both inner loops of Algorithm 1 and Algorithm 2 are based on the gradient projection method. In practice, we can use the variable
steps of gradient projection (Kelly et al., 1998; Zhu et al., 2008) to speed up the convergence of the inner loop. Taking (12) as an example, we update $y_r$, $r \in \mathcal{R}$,

$$y_r(t+1) = \Pi_{R_r} [y_r(t) + \alpha \gamma(y_r(t))]$$

where $\gamma \gamma(y_r(t)) = \frac{y_r(t)}{\partial \Omega U_r(y_r(t))}$ only depends on local information $y_r(t)$, which does not introduce difficulty to implementation.

4.3 Joint multi-path routing and retransmission control

**Algorithm 3: Successive Alternating Convex Approximation (SACA)**

**Require:** $\omega > 0$, $\zeta > 0$, $\gamma > 0$, $\alpha(1)$, $m = 1$, $v \in \mathcal{V}$.

1: repeat
2: For given $\alpha(m)$, run Algorithm 1 and set $y(m) = y^*$.
3: For given $y(m)$, run Algorithm 2 and set $\alpha(m) = \alpha^*$.
4: $U(m) = \tilde{U}(\alpha(m))$, $m = m + 1$.
5: until convergence.
6: Output $(y^*, \alpha^*) = (y(m), \alpha(m))$

In this sub-section, first we assume replicator factors are fixed so that the EUO problem reduces to an energy-aware multi-path routing problem. The optimal flow rates to this sub-problem are then fed to the other convex sub-problem, the MAC layer retransmission control problem. Similarly, the optimal replicator factors of retransmission control problem are then taken as the input to the multi-path routing problem. This alternating procedure is conducted as Algorithm 3 and the resulting sequence of optimal solutions to convex sub-problems, $(y(m), \alpha(m))$, is shown to converge to a steady-state point in the following Theorem 2.

**Theorem 2:** If the initial convex sub-problem (7) is feasible, then sequences $|U(m)|$ and $(y(m), \alpha(m))$ in Algorithm 3 converge to some $U^* \subseteq U$ and $(y^*, \alpha^*)$, respectively.

**Proof:** Let $m$ be arbitrary and suppose that $y(m)$ and $r \in \mathcal{R}$ respectively solve problem (7) and problem (14) in the $m$th iteration of Algorithm 3. The idea is to show that the sequence $|U(m)|$ is monotone increasing and upper bounded. From the problem formulations (7) and (14), we conclude that

$$\tilde{U}(\alpha(m-1)) \leq \tilde{U}(y(m)) \leq \tilde{U}(\alpha(m)) \leq \tilde{U}(y(m+1))$$

which implies the monotonicity of $|U(m)|$. It is also obvious that the feasible set $\mathcal{B}$ is closed since the utility function $U(c, y)$ is continuous on $\mathcal{B}$, which implies that $U(c, y)$ is bounded from above. This together with the monotonicity proves the convergence of sequence $|U(m)|$.

Meanwhile, the sequence $(y(m), \alpha(m))$, i.e., the counterpart of $|U(m)|$, is also convergent because $(\gamma(m), \alpha(m))$ is the unique solution achieving $U(m)$. This completes the proof.

**Comment 4:** In practice, replicator factors $\alpha_l$, $l \in \mathcal{L}$, should be integers, while the outputs of Algorithm 3, $\alpha^*_l$, $l \in \mathcal{L}$, are usually real numbers in $[\alpha^\text{min}_l, \alpha^\text{max}_l]$. To solve this conflict, we implement the optimised replicator factors $\alpha^*_l$ in a random way:

$$\alpha_l = \begin{cases} \lfloor \alpha^*_l \rfloor + 1, & \text{if } \text{rand}(0,1) \leq \frac{\lfloor \alpha^*_l \rfloor - \lfloor \alpha^*_l \rfloor}{\lfloor \alpha^*_l \rfloor} \leq 1, \forall l \in \mathcal{L}. \\ \lfloor \alpha^*_l \rfloor, & \text{otherwise} \end{cases}$$

In this way, we can simply verify that

$$E(\alpha_l) = (\lfloor \alpha^*_l \rfloor - \lfloor \alpha^*_l \rfloor) \times (\lfloor \alpha^*_l \rfloor + 1) + (1 - \lfloor \alpha^*_l \rfloor - \lfloor \alpha^*_l \rfloor) \times \lfloor \alpha^*_l \rfloor = \alpha^*_l$$

where $E(\alpha_l)$ denotes the mathematical expectation of $\alpha_l$.

5 WirelessHART-based WSNs

5.1 A brief overview of WirelessHART

WirelessHART is the first open wireless sensor-actuator network standard for industrial process monitoring and control that requires reliable and real-time data communication between sensor and actuator devices (HARTCOMM, 2007). As illustrated in Figure 1, a WirelessHART network includes the following basic elements:

1. **Field Devices** connected to the process equipment. All field devices are able to source, sink and forward packets on behalf of other devices in the network.
2. **Gateways** enabling communication between host applications and field devices. All data must pass through the gateway, which may have more than one access point.
3. **A Network Manager** responsible for configuring the network, health monitoring, managing routing tables and scheduling communication between devices.

![Figure 1](image)

**Figure 1** The WirelessHART network infrastructure (HARTCOMM, 2007) (see online version for colours)

Salient features of a WirelessHART network related to this paper include centralised network management, multi-channel TDMA transmission, and redundant routes, etc.
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- **Centralised network management**: Network-wide time synchronisation with 1 ms accuracy is supported. To guarantee timely and reliable data delivery, routing topology and transmission schedule are centrally computed at the network manager, and then disseminated to all devices in the network.

- **Multi-channel TDMA transmission**: The WirelessHART physical layer is compliant with the IEEE 802.15.4-2006 standard, supporting only 16 physical channels in the 2.4 GHz ISM band. Channel reuse is not allowed, namely, parallel transmissions scheduled in the same time slot must use different channels.

- **Redundant routes**: Each field device is configured with two routes: primary route and backup route. The backup route will heal the network failure in case the primary route fails.

5.2 Implementation of SACA

The SACA is essentially a centralised algorithm, which is usually not welcomed in practice. However, for WirelessHART-based WSNs, SACA fits perfectly because of the existence of the network manager responsible for the centralised resource management. A toy example is given to basically describe how the WirelessHART-based WSN works (see Figure 2). The network manager together with gateways acts as the sink node, and field devices correspond to sensor nodes. In Figure 2, two routes and TDMA scheduling of five logical links are configured by the network manager. The beacon frame is transmitted in the first slot of each superframe. The rest fifteen time slots are allocated among these five links. The duration of each time slot is set long enough to transmit a data packet and receive an ACK. A source node will receive an ACK if its data packet is correctly received by its destined receiver. In this paper we assume that ACK packets are intact. For energy conservation, retransmission stops and sleep mode starts as soon as the source receives an ACK.

From Figure 2, we know that the centralised management is realised through the periodic broadcast of beacons. Beacons are used to synchronise the attached devices and to describe the structure of superframes. However, for the implementation of SACA, beacons have to do more than the basic management. In Algorithms 1 and 2, the sink node broadcasts the collected energy information and reliability information on each route. Correspondingly, in implementation network manager recovers the received data and abstracts the centralised information from the data at the end of superframe. Then these abstracted ingredients are piggy-backed on the next beacon. Source nodes obtain the necessary components of updating flow rates (Algorithm 1) and dual variables (Algorithm 2) by reading the beacon. Then each source transmits its data packets with newly updated replicator factors and dual variables piggy-backed within its allocated time slots. The whole procedure keeps on going until convergence. Finally, based on the optimised result (flow rates and replicator factors), the network manager reallocates time slots and flow rates among links and routes, respectively.

Figure 2  Principle of WirelessHART-based WSNs

6 Simulation

In this section, Algorithms 1, 2 and 3 are simulated over the network as shown in Figure 3. Ten sensor nodes and one sink node are randomly located in the square $150 \times 150$ m$^2$. Each source node is configured with two routes. Sensor 5 and 6 act as both relay and source nodes. Here, we use the energy dissipation parameters in Zhu et al. (2008), where $\varepsilon_r=50$ nJ/bit, $\varepsilon_n=50$ nJ/bit, $\mu=50$ nJ/bit, $\eta=0.0013$ pJ/bit/m$^4$, and path loss factor $N=4$. Each node is charged the same energy price $\lambda_v=1.0 \times 10^{17}$, $v \in V$. The source rates are $s_v=4$ kbps, $v \in V$ and the weight factors $\omega_r=1, r \in R$. The channel error rate is set as $p=0.1$. Without loss of generality, utility functions are set as: $U_r(x)=\frac{1}{2x^2}$ and $W_r(x)=\frac{x^2}{2}$ ($\beta=3$).

First, we validate the effectiveness of Algorithms 1 and 2. In this section, we adopt the variable stepsize gradient projection algorithm to speed up the inner loop in Algorithms 1 and 2. The convergence results of Algorithms 1 and 2 are shown in Figure 4 and Figure 5, respectively. The common feature of these two figures is the fast rate of convergence. Though dual variables and physical variables oscillate slightly at beginning, two figures converge within 40 iterations.
Then, we validate the convergence of Algorithm 3. The initial replicator factors are chosen to be three. Based on the procedure in Algorithm 3, we generate two sequences \{\( \hat{U}(\alpha(m)) \)\} and \{\( \hat{U}(y(m)) \)\}, where \( \hat{U}(\alpha(m)) \) corresponds to the red ‘\( \Box \)’ sequence and \( \hat{U}(y(m)) \) corresponds to the blue ‘\( \Delta \)’ sequence in Figure 6. We could observe that in Figure 6 the utility sequence is monotone increasing, which validates the formula (26). Though no theoretical results guarantee the rate of convergence for Algorithm 3, in Figure 6 only six iterations are needed for convergence, i.e., three in Algorithm 3.

7 Conclusion

This paper has formulated a novel EUO problem for joint multi-path routing and MAC layer retransmission control in TDMA-based WSNs. A heuristic method called SACA, has been proposed to approximatively solve the non-convex EUO problem. The SACA decomposes the EUO problem into two alternating convex sub-problems. Both of these two sub-problems, i.e., energy-aware multipath routing and energy-aware retransmission control, have been shown to be solved by means of the dual decomposition method. These two alternating sub-problems are solved recursively until the sequence of optimal solutions converges to some steady-state point that approximates a solution to the EUO problem. We also talk about the feasibility of the implementation of SACA in WirelessHART-based WSNs.

Future work may generalise this work by incorporating more complex channel models into the EUO problem, such as the Gilbert-Elliott channel or fast fading channels. In this case, the EUO problem is essentially a stochastic optimisation problem. New mathematical tools, such as Markovian decision process or ergodic stochastic optimisation, are needed to deal with stochastic channels.

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References


Note

1 A vector of resources \( \mathbf{x} \) is \( \beta \)-proportionally fair if it is feasible and for any other feasible vector \( \mathbf{y} \), \( \sum \frac{x_i - y_i}{y_i} \leq 0 \).

When \( \beta = 1 \), \( \beta \)-proportional fairness reduces to the proportional fairness; when \( \beta = 2 \), then the harmonic mean fairness; when \( \beta \to \infty \), \( \beta \)-proportional fairness approximates the max-min fairness (Mo and Walrand, 2000).