Joint Routing and Power Control in Rayleigh-Faded Wireless Networks with ARQ Protocols

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Abstract—In this paper we formulate a goodput-oriented utility optimization problem for routing and power control in Rayleigh faded wireless networks with Automatic Retransmission reQuest (ARQ) protocols. This work proposes two heuristic approaches to estimate the goodput capacity in such wireless networks. The resulting approximated capacities are non-convex functions of power variables. As a result, the utility optimization problem is nonconvex, and for this we address the problem by solving a sequence of convex approximation problems. If the initial convex approximation is feasible, the sequence is shown to converge to a Karush-Kuhn-Tucker(KKT) point of the original utility optimization problem. The convex approximation problems are solved recursively by means of primal-dual methods that are shown to be amenable to distributed implementation by adjoint network. The seamless interaction between the successive convex approximation and the primal-dual algorithm constitutes the proposed successive primal-dual convex approximation (SPDCA) algorithm.

I. INTRODUCTION

Routing and power control are vital mechanisms for resource allocation in wireless networks [1]-[3]. In this paper, we consider a Rayleigh-faded wireless network with an established network topology and per-flow queuing in which the packets are passed from node to node to their destinations according to node-based routing algorithms. Detections of outage will trigger ARQ to combat the Rayleigh fading channel. In this case transmission rate is not any more equal to the reception rate. A new quantity named goodput is usually introduced to describe the actual rate of correctly transmitted bits [4]. Our focus is then on the problem of a joint optimization of transmit powers and flow rates to maximize network utility functions of goodput.

Former work [1]-[3] mainly focus on the resource allocation problem in wireless networks with error-free communications. However, error-free communication is impossible in fast fading environment since perfect channel knowledge is not available at both the transmitter and receiver due to the instantaneous feedback requirement for fast fading channel. References [5]-[6] capture the effects of erroneous nature of wireless channel and propose the rate-reliability tradeoff in wireless networks. In [7] and [8], the rate-outage constraint per link is introduced to cope with the fast fading(Rayleigh) channel. By doing so, the feedback requirement of channel state is unnecessary, but the network still suffers a little amount of fading-introduced outage.

ARQ protocol is adopted in this paper to compensate the outage in Rayleigh fading channel. The problem of joint routing and power control in wireless networks is in general difficult to solve because of the lack of convexity and separability due to the presence of interference. The introduction of Rayleigh fading channel and ARQ protocols further aggravates the difficulty in solving the optimization problem.

The first challenge is how to compute the goodput capacity for each link. The goodput capacity, also known as the maximum goodput [4], is defined as the maximal product of the physical layer transmission rate and successful transmission probability(STP) over a feasible set of transmission rates called the feasible rate region. For Rayleigh fading channel, STP has a closed form solution expressed in terms of transmission rate and power allocation. However, the power variables and rate variable are highly coupled, and thus an optimal transmission rate is difficult to calculate. We adopt two heuristic approaches to compute the goodput capacity. One is that we uniformly choose a rate from the feasible set. The other is that we find a lowerbound expression for STP, which has been proven to be tight in [8]. The adaptive transmission rate, Lambert W function [9] of ‘average SIR’, is also achieved correspondingly for the relaxed lowerbound of goodput capacity.

The second challenge is that the resulting goodput capacities are still inseparable and nonconcave with respect to power variables no matter which heuristic is adopted. This paper addresses the problem by combining a successive convex relaxation/approximation with primal-dual methods. More precisely, the proposed non-convex routing and power control problem is first approximated by a convex problem that is solved using primal-dual methods. The optimal solution is used to improve the approximation by stating and solving a new convex approximation problem. This procedure is repeated and the resulting sequence of optimal solutions of the convex approximation problems is shown to converge to

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a KKT point of the original problem, provided that the initial convex problem is feasible.

The third challenge results from the need for distributed implementation. As shown, once update in the primal-dual algorithm needs the whole information among the interfering links. Of course, excessive message passing mechanism, such as the "flooding" in [2], is not welcomed in practice. With the use of so-called adjoint network [1], we can implement the primal-dual algorithm in a distributed manner.

II. MODELING AND PROBLEM STATEMENT

A. Network and Communication Model

Consider a wireless network with an established network topology, in which all links share a common channel. We use \( \mathcal{N} := \{1, \ldots, N\} \) to denote the set of nodes. A number of flows compete for access to the wireless links without scheduling. One hop flows associated with logical links are called users. We use \( \mathcal{K} = \{1, \ldots, K\} \), \( \mathcal{F}(n) \), and \( \mathcal{K}(n) \) to denote the set of all logical links, the set of logical links incoming to node \( n \in N \), and the set of logical links originating at node \( n \in N \), respectively. The logical links are labeled in such a way that, for any two nonempty sets \( \mathcal{K}(n), \mathcal{K}(m) \subset \mathcal{K} \) with \( 1 \leq n < m \leq N \) and \( a \in \mathcal{K}(n), b \in \mathcal{K}(m) \), there holds \( a < b \). Finally, we assume that all nodes are synchronized and the time is divided in slots.

Suppose that \( \mathcal{W} \) is a collection of communication sessions and each session is identified by its unique source-destination node pair. For any session \( w \in \mathcal{W} \), let \( O(w) \) and \( D(w) \) denote its origin and destination nodes, respectively. The average rate of each session, say \( w \), is fixed and denoted by \( \lambda(w) \). Let the goodput flow variable of session \( w \) on link \( k \) be denoted by \( f_k(w) \) and let \( f = (f_k(w))_{w \in \mathcal{W}, k \in \mathcal{K}} \) be the link goodput vector. Then, by the flow conservation law, we have

\[
\forall w \in \mathcal{W} \forall n \in \mathcal{N} / (D(w)) \left( \lambda(w) \right) I_n(w) = \sum_{k \in \mathcal{K}(n)} f_k(w) - \sum_{k \in \mathcal{F}(n)} f_k(w)
\]

where \( I_n(w) := 1(n = O(w)) \) and \( 1(\cdot) \) is the indicator function.

In addition, the total goodput cannot exceed the corresponding goodput capacity \( c_k(p), k \in \mathcal{K} \). The goodput capacity constraint for link \( k \) yields

\[
\sum_{w \in \mathcal{W}} f_k(w) \leq c_k(p), k \in \mathcal{K}
\]

where \( p_k \geq 0 \) denotes a transmit power of user \( k \in \mathcal{K} \) and \( p := (p_1, \ldots, p_K) \geq 0 \) is a vector of all transmit powers, referred to as the power vector. The goodput capacity function of link \( k \), \( c_k(p) \), will be discussed in Section II. B in detail.

Each node, say node \( n \in \mathcal{N} \), is constrained on total power \( P_n > 0 \). This means that

\[
p \in \mathcal{P}, \mathcal{P} := \mathcal{P}_1 \times \ldots \times \mathcal{P}_N
\]

where \( \mathcal{P}_n := \{ x \in \mathcal{K}(n) : \sum_{k=1}^{\mathcal{K}(n)} x_k \leq P_n \} \) and \( \mathcal{P} \times \mathcal{P} \) is the Cartesian product of \( \mathcal{P}_n \) and \( \mathcal{P}_m \). In addition, there may be some hardware or regulatory limitations that impose individual constraints on link transmit powers and flow rates.

These constraints will be captured by maximizing our objective function over the following set \( \mathcal{B} := \{(p, f) : 0 \leq p_k \leq p_{\text{max}}(w) f_k \leq f_k \leq c_k(w), k \in \mathcal{K}, w \in \mathcal{W}\} \).

B. Goodput Capacity

We assume a Rayleigh-Rayleigh fading environment where both desired signals and interference signals at the receivers are subject to Rayleigh fading. In the Rayleigh-Rayleigh fading environment, the goodput capacities \( c_k(p) \), \( k \in \mathcal{K} \) are dictated by its transmission rate \( \mu_k \) and

\[
SIR_k(p) := \frac{V_k F_k p_k}{\sum_{l=1, l \neq k} V_k F_k p_l + \sigma_k^2}
\]

The above notion is defined as follows: \( \sigma_k^2 > 0 \) is the variance of an additive zero-mean Gaussian noise. \( V_k F_k p_k \geq 0 \) represents the path gain (not including fading) from transmitter \( l \in \mathcal{K} \) to receiver \( k \in \mathcal{K} \). In the same way, \( F_k F_k p_k \geq 0 \) represents the STP of link \( k \) when transmission rate is \( \mu_k \) and \( \mathcal{A} \) denotes the set of all possible transmission rates. Obviously, \( c_k(p) \) depends on the statistics of wireless fading channel, power and rate control. Following we propose two methods for estimating \( c_k(p) \).

1) Fixed Transmission Rate: In this case, each link \( k, k \in \mathcal{K} \) transmits according to a fixed rate \( \mu_k > 0 \) that is uniformly chosen from \( \mathcal{A} \). In Rayleigh-Rayleigh fading environment, the power received at the receiver of link \( k \) from the transmitter of link \( l \) is also an exponentially distributed random variable with mean value. If the transmission rate is \( \mu_k \geq 0 \), then the goodput capacity function of link \( k \) is given by

\[
c_k(p) = \mu_k e^{-\frac{\gamma_k}{\mu_k p_k}} \prod_{l \neq k}(1 + \frac{\gamma_k V_k F_k p_l}{V_k F_k p_k})^{-1}
\]

where STP takes the form [7]: \( Pr(SIR_k(p) \geq \gamma_k) = e^{-\frac{\gamma_k}{\mu_k p_k}} \prod_{l \neq k}(1 + \frac{\gamma_k V_k F_k p_l}{V_k F_k p_k})^{-1} \).

2) Adaptive Transmission Rate: In this case, we consider a lower bound on STP given by [8] to approximate it

\[
Pr(SIR_k(p) \geq \gamma_k) \geq \exp(-\frac{\gamma_k}{SIR_k(p)})
\]

where \( SIR_k(p) = \frac{V_k F_k p_k}{\sum_{l=1}^{\mathcal{K}} V_k F_k p_l + \sigma_k^2} \) is called 'average SIR' in [8]. Then the goodput capacity is underestimated as

\[
c_k(p) = \max_{\mu_k \in \mathcal{A}} g_k(\mu_k, p), \quad g_k(\mu_k, p) = \mu_k \exp(-\frac{\gamma_k}{SIR_k(p)})
\]

Let the partial derivative of \( g_k(\mu_k, p) \) with respect to \( \mu_k \) be zero, namely, \( \frac{\partial g_k(\mu_k, p)}{\partial \mu_k} = \exp(-\frac{\gamma_k}{SIR_k(p)})(1 - \frac{\mu_k p_k}{SIR_k(p)}) = 0 \). From this we obtain the optimal implicit solution of (6), \( \mu_k = W(SIR_k(p)) \), where \( W(\cdot) \) is the Lambert W function.

The Lambert W function, \( W(x) \), is defined to be the multivalued inverse of the function \( f(x) = xe^x \) [9]. \( W(x) \) is positive for \( x > 0 \) and is strictly concave.
With this in hand, the optimal goodput capacity yields
\[ c_k(p) = W(\text{STR}_k(p)) \exp\left( \frac{1}{\text{STR}_k(p)} - \frac{1}{W(\text{STR}_k(p))} \right). \tag{7} \]

**C. Optimization Problem**

In this paper, we take into account the Rayleigh fading channel and ARQ protocols by incorporating the goodput capacity into the traditional NUM framework to study the resource allocation problem over a joint space of transmit powers and routes:

\[ U = \max_{(p,f) \in \mathcal{P}} U(f) \quad \text{s.t.} \quad (1), (2), (3), (4) \text{ or } (7) \tag{8} \]

where \( U : \mathcal{R} \rightarrow \mathcal{R} \) is a utility function defined to be

\[ U(f) = \sum_{k \in \mathcal{K}} U_k(f_k) = \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \omega_k^{(w)} \Phi(f_k^{(w)}). \tag{9} \]

Here and hereafter, \( U_k \) is the utility function associated with link \( k \in \mathcal{K} \). The utility function of rates \( \Phi : \mathcal{R} \rightarrow \mathcal{R} \) is assumed to be differentiable, increasing and strictly concave. The weight vector \( \omega = (\omega_{(k)}^{(w)}, \ldots, \omega_{(k)}^{(w)}), w \in \mathcal{W} \), is any fixed positive vector, which can for instance be chosen according to the back-pressure policy [3]. We assume that \( \Phi(x) \rightarrow -\infty \) and \( \Phi'(x) \rightarrow +\infty \) as \( x \rightarrow 0 \) which implies that if \( (p^*, f^*) \) is a solution to (8), then \( (p^*, f^*) \) is a positive solution. Since an optimal power vector is positive, we can reformulate the problem using \( s := \log(p), p \in \mathcal{P} \cap \mathcal{P}^+ \), where the logarithm is taken elementwise.

**III. SUCCESSIVE CONVEX APPROXIMATION METHOD**

**A. Utility Maximization with Fixed Transmission Rate**

In this section, we will start with the utility maximization problem (8) with fixed transmission rate (4). Taking the logarithm on all sides in (2) yields

\[ d_k(s,f) := \log\left( \sum_{w \in \mathcal{W}} f_k^{(w)} \right) + \sum_{l \neq k} \log(1 + \frac{\gamma V_{lk}}{V_{kk}} e^{n_l - n_k}) + \frac{\gamma \sigma_k^2}{V_{kk}} e^{-n_k} - \log(c_k) \leq 0. \tag{10} \]

All terms in (10) are convex except the first term \( \log(\sum_{w \in \mathcal{W}} f_k^{(w)}) \), while in contrast, \( \log(\sum_{w \in \mathcal{W}} f_k^{(w)}) \) is concave in \( f_k = (f_k^{(w)})_{w \in \mathcal{W}} \). As a result, the problem is difficult to solve and we approximate it by a sequence of convex approximations indexed by \( t \in \{0,1,2,\cdots\} \). These approximations are based on the fact that any concave function can be tightly upper bounded by an affine function [10, page 70]. More precisely, for the \( t \)-th approximation, we choose some feasible rates \( f_k^{(w)}(t) \neq 0 \) \((k \in \mathcal{K}, w \in \mathcal{W})\), and bound from above the first term in (10) as follows

\[ \log\left( \sum_{w \in \mathcal{W}} f_k^{(w)} \right) \leq a_k^T(t) f_k + b_k(t) \tag{11} \]

where \( a_k^T(t) = \frac{1}{\sum_{w \in \mathcal{W}} f_k^{(w)}(t)}(1,\cdots,1) \) and \( b_k(t) = \log(\sum_{w \in \mathcal{W}} f_k^{(w)}(t)) - 1 \). Now, using the upperbound (11) in (10), the \( t \)-th convex relaxation of (8) with fixed transmission rate takes the form

\[ \hat{U}(t) := \max_{(s,f) \in \hat{\mathcal{R}}} U(f) \quad \text{s.t.} \quad (1), (3), (7) \tag{12} \]

\[ \hat{d}_k(s,f) := a_k^T(t) f_k + b_k(t) + \frac{\gamma \sigma_k^2}{V_{kk}} e^{-n_k} + \sum_{l \neq k} \log(1 + \frac{\gamma V_{lk}}{V_{kk}} e^{n_l - n_k}) \tag{13} \]

where \( \hat{\mathcal{R}} := \{(s,f) : e^k \leq p_k^{\text{max}}, j_k^{(w)} \leq f_k^{(w)} \leq \hat{f}_k^{(w)}, k \in \mathcal{K}, w \in \mathcal{W}\} \).

We start the sequence of convex approximations with \( t = 0 \) and compute \( \hat{d}_k(0), b_k(0) \) for any feasible rates \( f_k^{(w)}(0) \neq 0 \) \((k \in \mathcal{K}, w \in \mathcal{W})\). Once the \( t \)-th convex problem is solved, we use the resulting rates \( \tilde{f}(t) \) to update

\[ a_k^T(t + 1) = 1 / \sum_{w \in \mathcal{W}} \tilde{f}_k^{(w)}(t)(1,\cdots,1), \]

\[ b_k(t + 1) = \log(\sum_{w \in \mathcal{W}} \tilde{f}_k^{(w)}(t)) - 1 \tag{14} \]

and solve the \( (t + 1) \)-th problem (12)–(13). We repeat this process until some convergence criteria are satisfied.

**Theorem 1:** If the initial convex approximation is feasible, then the sequences \( \{\hat{U}(t)\} \) and \( \{\hat{s}(t), \hat{f}(t)\} \) converge to some \( \hat{U}^* \leq \hat{U} \) and \( (\hat{s}^*, \hat{f}^*) \), respectively, where \( (\hat{s}^*, \hat{f}^*) \) is a KKT point of original problem (8).

**Proof:** Due to the lack of space, we only give the sketch of this proof. We will show that the sequence \( \{\hat{U}(t)\} \) is non-decreasing and upper bounded. Further, we can easily verify that \( \hat{d}_k(s,f) \) and \( \hat{d}_k(s,f) \) satisfy the Theorem 1 of [11], which means \( \{\hat{s}(t), \hat{f}(t)\} \) is a KKT point of original problem (8).

**B. Utility Maximization with Adaptive Transmission Rate**

In this section, we will proceed with the utility maximization problem (8) with adaptive transmission rate (7). Taking the logarithm on all sides in (2) yields

\[ \log\left( \sum_{w \in \mathcal{W}} f_k^{(w)} \right) - \frac{1}{\text{STR}_k(e^s)} + G_k(s) < 0. \tag{15} \]

where \( G_k(s) = \frac{1}{W(\text{STR}_k(e^s)))} + \log\frac{1}{W(\text{STR}_k(e^s)))} \).

According to Lemma 1 in [1], the logarithm of inverse SIR

\[ h_k(s) := \log(1/\text{STR}_K(e^s)), 1 \leq k \leq K \tag{16} \]

is a convex function of \( s \in \mathcal{R}^K \). An immediate conclusion is that \( 1/\text{STR}_K(e^s) \) is a convex function since log-convexity implies convexity. Thus, the first two items in (15) are concave. In contrast, the third term \( G_k(s) \) is a convex function of \( s \) as shown by the following theorem.

**Theorem 2:** \( G_k(s) \) is convex on \( \mathcal{R}^K \), i.e.,

\[ G_k(\tilde{s}(\mu)) \leq (1 - \mu) G_k(\tilde{s}) + \mu G_k(\hat{s}) \tag{17} \]

for all \( \tilde{s}, \hat{s} \in \mathcal{R}^K \) and \( \mu \in [0,1] \).

**Proof:** According to the definition of Lambert W function, \( x = W(x)e^{W(x)} \), we differentiate both sides of it to obtain

\[ dW(x)/dx = 1/(e^{W(x)}(1 + W(x))). \tag{18} \]
Further, based on (18), it is easy to prove that both $\log \frac{1}{W(e^t)}$ and $\frac{1}{W(e^t)}$ are strictly decreasing and convex with respect to $x$.

We define the logarithm of SIR as $g_k(s) = -b_k(s)$ which is a concave function of $s \in \mathbb{R}^k$. Let $\hat{s}, \bar{s} \in \mathbb{R}^K$ with $\hat{s} \neq \bar{s}$ be arbitrary. For all $\mu \in [0, 1]$, we have

$$G_k(s(\mu)) = \frac{1}{W(\sum_{r \in \mathcal{R}} e^{-\mu s^r})} + \log W(\sum_{r \in \mathcal{R}} e^{-\mu s^r}) \leq a \frac{1}{W(\sum_{r \in \mathcal{R}} e^{-\mu s^r})} + \log W(\sum_{r \in \mathcal{R}} e^{-\mu s^r})$$

Further, based on (18), it is easy to prove that both $\log \frac{1}{W(e^t)}$ and $\frac{1}{W(e^t)}$ are strictly decreasing and convex with respect to $x$.

In the following, we still use a sequence of convex approximations to approximate the problem (8). Similar to (11), for the $r$th approximation, we use an affine function to bound from above the convex function as follows: for some feasible solutions $f_k^{(w)}(t) \neq 0, s \in \mathbb{R}^K(k \in \mathcal{K}, w \in \mathcal{W})$,

$$\log \left( \sum_{w \in \mathcal{W}} f_k^{(w)}(t) - \frac{1}{\sum_{w \in \mathcal{W}} e^{-\mu s^r(t)}} \right) \leq a_k(t) \bar{a}_k(t) + b_k(t)$$

and

$$\bar{b}_k(t) = - \frac{1}{\sum_{w \in \mathcal{W}} e^{-\mu s^r(t)}} - \bar{a}_k(t) s(t).$$

Then, using the above convex upperbound in (15), the $r$th convex relaxation of (8) with adaptive transmission rate takes the form

$$\hat{U}(t) := \max_{(s,f) \in \mathcal{S}} U(f) \quad \text{s.t.} \quad (1), (3), (19)$$

Analogous to last subsection, we repeat the successive convex approximation process until convergence is achieved.

Theorem 3: Suppose that $(\hat{s}(t), \hat{f}(t))$ solves (19)–(20). If the initial convex approximation is feasible, then the sequences $\{\hat{U}(t)\}$ and $\{|\hat{s}(t), \hat{f}(t)|\}$ converge to some $\hat{U}^* \leq U$ and a KKT point of original problem (8), $(s^*, f^*)$, respectively.

IV. DISTRIBUTED IMPLEMENTATION

Due to the limited space, we just devise a primal-dual algorithm to solve problem (19)-(20). Let $\nu = (\nu_k^{(w)})$, $\nu = (\nu_k^{(n)})$, $\varepsilon = (\varepsilon_n)_{n \in \mathcal{N}}$, $\varepsilon$ = (\(\varepsilon_{n}^{(w)})_{n \in \mathcal{N}}$), and $\theta = (\theta_k)_{k \in \mathcal{K}}$ be Lagrange multipliers for constraints (1), (3), and (20), respectively. The physical constraints $\mathcal{B}$ are satisfied implicitly. For brevity, let $\Gamma$ and $\Lambda$ denote the set of original variables $(s, f)$ and the set of dual variables $(\nu, \varepsilon, \theta)$, respectively. We define the feasible set $\Omega = \mathcal{R}^{\nu} \times \mathcal{R}^{\varepsilon} \times \mathcal{R}^{\theta}$ of $\Lambda$. The Lagrangian function associated with problem (19) is given for some $t \in \{0, 1, 2, \ldots\}$

$$L(\Gamma, \Lambda) = U(f) - \sum_{n \in \mathcal{N}} \varepsilon_n \left( \sum_{k \in \mathcal{K}} \varepsilon_k + P_n \right)$$

and

$$\sum_{n \in \mathcal{N}} \sum_{w \in \mathcal{W}} \nu_k^{(w)}(\lambda(w)1_n^{(w)} - \sum_{k \in \mathcal{K}} f_k^{(w)} + \sum_{k \in \mathcal{K}} f_k^{(w)})$$

$$- \sum_{k \in \mathcal{K}} \theta_k \left[ \left( \frac{a_k(t)}{\bar{a}_k(t)} \right) + b_k(t) + \bar{b}_k(t) + G_k(s) \right]$$

(21)

It is easy to show that the Lagrangian function (21) is a concave-convex function and strong duality holds. These observations imply that the KKT conditions are necessary and sufficient conditions for optimality. Thus, we can find the maximum in (19) by solving the KKT conditions. This is equivalent to finding a stationary point $(\Gamma^*, \Lambda^*)$ of the Lagrangian function (21). We apply the primal-dual algorithm to find a stationary point of $(\Gamma, \Lambda)$. In the $(m+1)$th iteration,

$$\left\{ \begin{array}{l}
\Gamma(m+1) = \Pi_\mathcal{R} \Gamma(m) + \delta \nabla_{\Gamma} L(\Gamma(m), \Lambda(m)) \\
\Lambda(m+1) = \Pi_\Omega \Lambda(m) - \delta \nabla_{\Lambda} L(\Gamma(m), \Lambda(m))
\end{array} \right. \quad (22)$$

where $\delta$ is a sufficiently small stepsize. $\Pi_\mathcal{R} \Lambda$ and $\Pi_\Omega \Gamma$ are the projections on $\Omega$ and $\mathcal{S}$, respectively. From (21), we compute partial derivatives of $L(\Gamma, \Lambda)$ with respect to $(\Gamma, \Lambda)$,

$$\frac{\partial L}{\partial f_k^{(w)}} = \omega_k^{(w)} \phi^{(f_k^{(w)})} - \frac{\theta_k}{\sum_{w \in \mathcal{W}} f_k^{(w)}} + \omega_k^{(r(\nu_k^{(w)}) - \nu_k^{(w)})}$$

$$\frac{\partial L}{\partial s_k} = -\varepsilon_k e_k - \frac{\theta_k}{\sum_{w \in \mathcal{W}} e^{-\mu s^r(t)}}$$

and

$$\frac{\partial L}{\partial \nu_k^{(w)}} = \sum_{k \in \mathcal{K}} f_k^{(w)} - \sum_{k \in \mathcal{K}} f_k^{(w)} - \lambda(w)1_n^{(w)}$$

$$\frac{\partial L}{\partial \varepsilon_n} = P_n - \sum_{k \in \mathcal{K}} e_k$$

$$\frac{\partial L}{\partial \theta_k} = -\frac{a_k(t)}{\bar{a}_k(t)} + b_k(t) - \bar{b}_k(t) - G_k(s)$$

(23)

and

$$(24)$$

Once the inner loop has converged, Theorem 1 ensures that the outer...
Algorithm 1 SPDCA

Require: \( \omega > 0, \sigma > 0, t = 0, m = 0, \Gamma(0), \Lambda(0), \) and \( \delta > 0. \)
Ensure: \((p, f)\)
1: repeat
2: Each transmitter updates affine parameters \((a_k(t), b_k(t), \tilde{a}_k(t), \tilde{b}_k(t))\) of the \( t \)th approximation.
3: repeat
4: Over Channel 1: Concurrent transmission of one data packet at transmit power \( e^{s_k(m)} \), \( k \in \mathcal{K} \), with the value of \( s_k(m) \) appended in the packet.
5: Over Channel 2: Concurrent transmission of pilot sequences at powers \( e^{s_k(m)} \), \( k \in \mathcal{K} \), with receiver-side estimation of \( \tilde{S}_m^k(e^{s_k(m)}) \).
6: All receivers feed necessary estimates and variables including \( \tilde{S}_m^k(e^{s_k(m)}) \) and \( v^{(w)}(m) \) back to the corresponding transmitters via per-link control channel.
7: Over Channel 2: Concurrent transmission in the adjoint network with transmitter-side estimation of the received power. Receivers with negative \( s_k(m) \) transmit first and then receivers with non negative \( s_k(m) \) transmit concurrently. The variance of the zero-mean input symbols is \( [e^{s_k(0)}]_{s_k(m)} \).
8: Transmitter side computation of \( \Gamma(m) \) and \( \Lambda(m) \).
9: \( m = m + 1 \).
10: until \( L(\Gamma(m), \Lambda(m)) - L(\Gamma(m - 1), \Lambda(m - 1)) < \sigma \).
11: \( (\tilde{s}(t), \tilde{f}(t)) = (s(m), f(m)) \).
12: \( t = t + 1 \).
13: until \( \tilde{U}(t) - \tilde{U}(t - 1) < \sigma \).
14: \( (p^*, f^*) = (\tilde{U}(0), \tilde{f}(0)) \).

loop converges to some KKT point \((s^*, f^*)\) of the original problem (8) as \( t \to \infty \).

V. SIMULATION

Consider a wireless network as in Fig. 1. There are six logical links, four nodes and two sessions \( \lambda(1 \rightarrow 4) = 0.3, \lambda(2 \rightarrow 4) = 0.3 \). The path gain \( V_{k,l}(k, l \in \mathcal{K}) \) were randomly chosen from \([0, 0.2]\). Moreover, we assume \( \omega = (1, 2, \lambda, 3, \sigma, \epsilon), \) and \( \Phi(x) = \log(x) \).

The convergence of Algorithm 1 is validated by Fig.2. Though there are no theoretical results regarding to the speed of convergence of successive convex approximation method, we can see that the optimal MAC-layer sequence \( \{\tilde{U}(t)\} \) in SPDCA converges within 10 times approximation from Fig.2.

ACKNOWLEDGMENT

This work was supported by the China National Funds for Distinguished Young Scientists(Grants No. 60725312), The Special Program for Key Basic Research Founded by MOST(Grant No. 2010CB334705), The Important National Science and Technology Specific Project(Grant No. 2010ZX03006-005-01), the Liaoning Provincial Natural Science Foundation of China (Grant No. 20092083), and by the German Research Foundation (DFG) under grant STA864/3-1.

This work was performed while Meng Zheng was with the Fraunhofer Heinrich Hertz Institute, Berlin, 10587 Germany.

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