1 INTRODUCTION

Significant progress in understanding the nature of gamma-ray bursts (GRBs) and their afterglows has been made in the recent decade (for reviews see Fishman & Meegan 1995; Piran 1999; van Paradijs, Kouveliotou & Wijers 2000; Cheng & Lu 2001; Mészáros 2002). However, this phenomenon is still a great mystery. One of the most difficult parts of solving this mystery is the central engines of GRBs, which are far away from emission regions and are hidden behind our observations (e.g. Cheng & Lu 2001). It is believed that GRBs are produced by conical ejecta powered by the central engines. Sharp breaks and/or quick decays in afterglow light curves are thought to be evidence for collimated ejecta (e.g. Rhoads 1999; Sari, Piran & Halpern 1999; Dai & Cheng 2001). Starting with GRB 990123, such evidence has been rapidly growing (e.g. Bloom, Frail & Kulkarni 2001, and references therein). In a previous work we showed that the current observations of prompt gamma-ray emissions and X-ray afterglow emissions satisfy the prediction of a GRB jet (Liang 2004). Both the dynamics and afterglow light-curve behaviours of GRB jets were investigated numerically (Panaitescu, Mészáros & Rees 1998; Huang et al. 2000; Moderski, Sikora & Bulik 2000; Granot & Kumar 2003; Kumar & Granot 2003; Salmonson 2003), and some GRB afterglows of interest were also fitted (Panaitescu & Kumar 2001a,b, 2002).

GRB jet structure models are currently under heavy debate. Two currently competing models are the universal structured jet (USJ) model (e.g. Mészáros, Rees & Wijers 1998; Dai & Gou 2001; Rossi, Lazzati & Rees 2002; Zhang & Mészáros 2002a; Granot & Kumar 2003; Kumar & Granot 2003; Panaitescu & Kumar 2003; Wei & Jin 2003) and the uniform jet model (e.g. Rhoads 1999; Frail et al. 2001). X-ray flashes (XRFs), which are thought to be the low-energy extension of typical GRBs (Kippen et al. 2003; Lamb, Donaghy & Graziani 2003; Sakamoto et al. 2004), seem to provide more jet structure signatures. If both GRBs and XRFs are the same phenomenon, any jet structure model should present a unified description for GRBs and XRFs. Zhang et al. (2004a) showed that the current GRB/XRF prompt emission/afterglow data can be described by a quasi-universal Gaussian-like (or similar structure) structured jet with a typical opening angle of $\sim 6\degree$ and with a standard jet energy of $\sim 10^{51}$ erg. Based on High Energy Transient Explorer 2 (HETE-2) observations, Lamb et al. (2003) proposed that the uniform jet model reasonably describes the unified scheme of GRBs/XRFs. Very recently, the two-component jet model was advocated by Berger et al. (2003a) based on the observations of GRB 030329, which has two different jet breaks in an early optical afterglow light-curve (0.55 d; Price et al. 2003) and in a late radio light-curve (9.8 d; Berger et al. 2003a). Millimetre observations further support the two-component jet in this burst (Sheth et al. 2003). Simulations of the...
propagation and eruption of relativistic jets in massive Wolf–Rayet stars by Zhang, Woosley & Heger (2004b) also showed signatures of the two-component jet. In a previous work, we found a bimodal distribution of the observed peak energy \( E_p \) of \( vF_\nu \) spectra of GRBs/XRFs, and concluded that the two-component jet model can explain this distribution (Liang & Dai 2004). Our results suggest that the two-component jet seems to be universal for GRBs/XRFs. Numerical calculations of such a model were also presented by Huang et al. (2004).

The USJ model has a predictive power. This might be served as a test for this model. Perna, Sari & Frail (2003) found that both the observational and theoretical probability distributions of \( \theta, P(\theta) \), are roughly consistent with each other. However, Nakar, Granot & Guetta (2004) showed that the theoretically two-dimensional probability distribution, \( P(\theta, \epsilon) \), poorly agrees with the observational one. We note that both Perna et al. (2003) and Nakar et al. (2004) came to conclusions from the comparison of the USJ model’s predictions with the current observations. Whereas, the current sample of GRBs with \( \theta \) available is very small (16 GRBs in their works). Observational biases and sample incompleteness affect greatly this sample. It is difficult to draw a robust conclusion in a statistical sense of GRBs with \( \theta \) available from the comparison of \( P(\theta) \) or \( P(\theta, \epsilon) \) derived from this sample with that predicted by the USJ model. In addition, both theoretical \( P(\theta) \) and \( P(\theta, \epsilon) \) are sensitive to instrument’s threshold setting. In Perna et al. (2003) and Nakar et al. (2004), the threshold is the same as CGRO/BATSE. However, most of the bursts with \( \theta \) available in the current GRB sample were not observed by CGRO/BATSE. This difference also affects greatly the comparison results. We therefore make a further test by simulations. Based on the standard energy reservoir of GRB jets Frail et al. (2001); Bloom, Frail & Kulkarni (2003); Panaitescu & Kumar (2001a); Piran et al. (2001); Berger, Kulkarni & Frail (2003b) and the relationship between \( E_p \) and \( E_{iso} \), the equivalent isotropic energy (Amati et al. 2002; Lloyd-Ronning & Ramirez-Ruiz 2002; Atteia 2003; Sakamoto et al. 2004; Lamb, Donaghy & Graziani 2003; Liang, Dai & Wu 2004; Yonetoku et al. 2004), we test the \( P(\theta) \) and \( P(\theta, \epsilon) \) predicted by the USJ model with simulations on observational bases for a detection threshold of \( S > 4 \times 10^{-5} \text{ erg cm}^{-2} \), where \( S \) is the total fluence in an energy band \( > 20 \text{ keV} \). We show that both \( P(\theta) \) and \( P(\theta, \epsilon) \) derived from simulations and from the USJ model are consistent with each other.

This paper is arranged as follows. The empirical model is presented in Section 2. Our simulation procedure is described in Section 3. The results of the comparison of our simulation results with that predicted by the USJ model are presented in Sections 4. Discussion and conclusion are presented in Section 5. Throughout this work we adopt \( H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( \Omega_\Lambda = 0.3 \), and \( \Omega_m = 0.7 \).

2 Model

Our simulation analysis model is based on the statistical results of a standard energy reservoir in GRB jets and the relationship between \( E_p \) and \( E_{iso} \). The energy release of a GRB jet in the gamma-ray band is given by

\[
\epsilon_{52} = \frac{\epsilon_{iso,52}}{E_p,52}(1 - \cos \theta),
\]

where \( E_{iso,52} = E_{iso}/10^{52} \text{ erg} \). The relationship between \( E_{iso} \) and \( E_p \) is given by

\[
E_{iso,52} = k(E_p,52(1 + z))^2,
\]

where \( k \) is a coefficient, and \( E_p,52 = E_p/10^2 \text{ keV} \). This empirical relation was further investigated in standard synchrotron/inverse-Compton/synchro-Compton models (Ramirez-Ruiz & Lloyd-Running 2002; Zhang & Mészáros 2002b). Sakamoto et al. (2004) and Lamb et al. (2003) pointed out that HETE-2 observations not only confirm this correlation, but also extend it to XRFs. In a recent work, we also proved that this relationship holds within a burst (Liang et al. 2004). From equations (1) and (2), we obtain

\[
\theta = \arccos \left( 1 - \frac{E_p}{kE_p,52(1 + z)^2} \right).
\]

Because the values of \( k \) and \( \epsilon_{52} \) can be obtained by the current GRB sample, the \( \theta \) value of a burst can be estimated by equation (3), if its \( E_p \) and \( z \) are available. If both distributions of \( E_p \) and \( z \) can be established by observations or theory, one can perform simulations to derive a GRB sample at the least observational bias, and obtain \( P(\theta) \) and \( P(\theta, \epsilon) \) from the simulated sample. Such an \( E_p \) distribution for a given detection threshold may be established by the BATSE observations. The redshift distribution is impossible to be modelled by the presented GRB sample because of various observational biases, such as GRB detection, localization, and redshift measurement (Bloom 2003). Please note that, in our analysis, we focus on the comparison of the simulated \( P(\theta) \) and \( P(\theta, \epsilon) \) with that predicted by the USJ model. If we use the same redshift distribution model in our simulations and in the USJ model, the comparison results are not significantly affected by the redshift distribution (for a detailed discussion in Section 5). Thus, we perform simulations to derive \( P(\theta) \) and \( P(\theta, \epsilon) \) and compare them to the predictions of the USJ model for a given detection threshold, assuming the GRB rate as a function of redshift is proportional to the star formation rate. We present the distributions of these parameters used in our simulations in the following.

The strong correlations shown in equations (1) and (2) indicate that both \( \epsilon_{52} \) and \( k \) have a standard candle feature. They should not be significantly affected by observational biases and the sample selection effect. Their distributions can be modelled by the present GRB sample. From a sample of GRBs with \( \theta \) available presented by Bloom et al. (2003), we find that the \( \epsilon_{52} \) distribution can be modelled by a lognormal one with a centre at \( -0.9 \) and a \( \sigma_{\log\epsilon_{52}} = 0.3 \), i.e.

\[
P(\log \epsilon_{52}) = P_{\epsilon,0}\exp\left(-\left[\log \epsilon_{52}+0.9\right]^2/0.3^2\right)
\]

where \( P_{\epsilon,0} \) is a normalized constant given by

\[
\int_{-\infty}^{\infty} P(\log \epsilon_{52}) d\log \epsilon_{52} = 1.
\]

From a GRB sample presented by Amati et al. (2002), we find that the \( k \) distribution can be modelled by

\[
P(k) = P_{k,0}\exp\left(-2(k-1)^2/0.4^2\right),
\]

where \( P_{k,0} \) is a normalized constant given by

\[
\int_0^\infty P(k) dk = 1.
\]

The observed \( E_p \) distribution is significantly affected by observational biases and sample selection effects, especially when the completeness at low fluxes is considered. We modelled the \( E_p \) distribution for a given detection threshold from GRB observations. In a previous paper, we studied the observed \( E_p \) distribution of GRBs and XRFs, combined with both HETE-2 and BATSE observations. We found that the observed \( E_p \) distribution for GRBs/XRFs is a bimodal one with peaks of ~30 and ~200 keV (Liang & Dai 2004). The ~30 keV peak has a sharp cut-off at its lower energy side because of the limitation of the current instrument thresholds. This bimodal distribution should be further examined by future observations. In addition, for some extremely soft XRFs, they violate the model used in our analysis (equation 3; e.g. Liang & Dai 2004). We therefore cannot use this \( E_p \) distribution in our analysis. It is well known that the observed \( E_p \) distribution of typical GRBs is narrowly clustered at 200–400 keV (e.g. Preece et al. 2000). The \( E_p \) distribution of bright GRBs has been well established. Thus, we consider
only bright GRBs in our analysis. Preece et al. (2000) presented the observed \(E_p\) distribution for a long, bright GRB sample selected by \(S > 4 \times 10^{-5}\) erg cm\(^{-2}\) in an energy band \(>20\) keV. We use this GRB sample to model the \(E_p\) distribution, which is

\[
P(\log E_{p,2}) = P_{E0} e^{-2(\log E_{p,2} - 0.38)^2/0.45^2},
\]

(6)

\(P_{E0}\) is a normalized constant, which is given by \(\int_{-\infty}^{\infty} P(\log E_{p,2}) d \log E_{p,2} = 1\).

The GRB rate as a function of redshift is assumed to be proportional to the star formation rate (e.g. Bromm & Loeb 2002). The SF1 model from Porciani & Madau (2001) is used in our analysis. Thus, the redshift distribution is modelled by

\[
P(z) = P_{z0} e^{3z/38 + 45},
\]

(7)

where \(P_{z0}\) is a normalized constant. Since the largest redshift of GRBs is 4.5, we restrict \(z \leq 4.5\), and \(P_{z0}\) is given by \(\int_0^{4.5} P(z) dz = 1\).

3 SIMULATION PROCEDURE

Based on the empirical model and its parameter distributions presented above, we simulate a sample including \(10^6\) GRBs. Each simulated burst is characterized by a set of \((k, \varepsilon_{52}, E_{p,2}, z)\), and it also satisfies our threshold setting. The threshold setting corresponds to the sample selection criterion of the sample in Preece et al. (2000), which is \(S > 4 \times 10^{-5}\) erg cm\(^{-2}\). Our simulation procedure is described as follows.

1. Derive the accumulative probability distributions of the parameters \(Q(x)\), where \(x\) is one of these parameters, from equations (4)–(6). In order to save calculation time, we use the discrete forms of these distributions. By giving a bin size and truncating \(x\) at a given value \(a\), we calculate \(P(x_i)\) by using equations (4)–(6), where \(x_i\) is the \(i\)th bin of parameter \(x\), and then derive \(Q(x_i) = \Sigma_{a=0} P(x_i)\). The normalized constants in equations (4)–(7) are given by \(Q(x_i)|_{x=0} = 1\).

2. Simulate a GRB. We first generate a random number \(m (0 < m < 1)\), and obtain the value of \(x\) from the inverse function of \(Q(x) = m\), i.e. \(x = Q^{-1}(m)\). Please note that \(Q(x)\) is in a discrete form. The \(x\) value for a given \(m\) is restricted by \(Q(x_i) < m\) and \(Q(x_{i+1}) < m\). Then, the \(x\) value is given by \(x = (x_{i+1} + x_i)/2\). Repeating this step for each parameter, we get a simulated GRB characterized by a set of \((k, \varepsilon_{52}, E_{p,2}, z)\). If this burst violates \(\varepsilon_{52}/k[E_{p,2}(1 + z)]^2 < 1\), it does not satisfy equation (3) and is ruled out without further consideration.

3. Examine whether or not the simulated GRB satisfies our threshold setting. For a simulated GRB, we calculate its flux in the observer frame by \(F = E_{\text{iso}}(1 + z)/4\pi D_L^2(z)T\), where \(D_L(z)\) is the luminosity distance at \(z\), \(T\) is the ‘effective’ duration of the burst, and \(E_{\text{iso}}\) is given by equation (2). The ‘effective’ duration in Perna et al. (2003) is 8 s. In our analysis we take \(T = 12\) s (for a detailed discussion see Section 4). The limit of \(F\) is given by \(F_{\text{lim}} = S/T = 3.3 \times 10^{-6}\) erg cm\(^{-2}\) s\(^{-1}\). If \(F > F_{\text{lim}}\), the burst is included into our mock GRB sample.

4. Repeat steps (2) and (3) to obtain a mock sample of \(10^6\) GRBs.

4 RESULTS

We calculate the \(\theta\) value for each mock GRB by equation (3). The relative probability of \(\theta\) is shown in Fig. 1 (the stepped line). For comparison, the results derived from the USJ model is also plotted in Fig. 1 (the straight line). Please note that, when we calculate the \(\theta\) distribution by the USJ model, the ‘effective’ duration \(T\) and the threshold setting are different from that used in Perna et al. (2003). \(T\) is an adjustable parameter, which is taken as 8 s in order to obtain a distribution that is agreement with the observational data in Perna et al. (2003). From Fig. 1, we can see that the simulated distribution is a lognormal one centering at \(\log \theta = -1.3\). This result is significantly different from the observational result.\(^1\) The mean of \(\theta\) for a sample of GRBs with \(\theta\) available in Bloom et al. (2003) is \(-0.12\) rad, while our simulation result is clustered at 0.04 rad. Hence, we do not take the same \(T\) value as that in Perna et al. (2003). We adjust its value to let both the simulated \(\theta\) distribution and that derived by the USJ model peak at the same position. We obtain \(T \sim 12\) s. For the threshold setting, we adopt \(F_{\text{lim}} = 3.3 \times 10^{-6}\), as we mentioned in the above section. The maximum redshift (\(z_{\text{max}}\)) up to which a burst with apparent angle \(\theta\) can satisfy the detection threshold is obtained from

\[
F_{\text{lim}} = \frac{E_{\text{iso}}}{4\pi D_L^2(z_{\text{lim}})(1 - \cos \theta)T}
\]

(8)

where \(E_{\text{iso}} = 1.33 \times 10^{51}\) erg (Bloom et al. 2003). As the maximum redshift in current GRB observations is 4.5, we set \(z_{\text{max}} = 4.5\) when \(z_{\text{max}} > 4.5\).

From Fig. 1, we find that the \(\theta\) distribution of the mock GRB sample range is 0.001–0.25 rad, and the two distributions are well consistent when \(\theta < 0.10\) rad, while the simulated \(\theta\) distribution is lower than that predicted by the USJ model when \(\theta > 0.10\) rad. We evaluate the consistency of these two distributions by a Kolmogorov–Smirnoff (K-S) test (Press et al. 1997, p. 617). The result of the K-S test is described by a statistic of \(P_{K-S}\): a small value of \(P_{K-S}\) indicates a significant difference between two distributions (\(P_{K-S} = 1\) indicates that two distributions are identical, and \(P_{K-S} < 10^{-4}\) suggests that the consistency of two distributions should be rejected; e.g. Bloom 2003). The K-S test shows that \(P_{K-S} = 0.73\), indicating that the consistency of the two distributions is acceptable.

Nakar et al. (2004) found that the two-dimensional distribution of the probability from observations, \(P(\log \theta, z)\), is poorly

\(^1\) Please note that the simulated \(\theta\) distribution is dependent on the star formation rate model and the detection threshold. It cannot be regarded as a true \(\theta\) distribution. Any comparison between this distribution and observational results is meaningless.
consistent with that predicted by the USJ model. We have mentioned in Section 1 that this discrepancy might be due to the threshold setting and sample incompleteness. We compare the two-dimensional distribution derived from our simulations (the grey contour plot) with that predicted by the USJ model, and the shaded areas, from light to dark grey, represent the same probability regions derived from our simulations. From Fig. 2, we find that our simulation result is consistent with that predicted by the USJ model. To clearly illustrate this consistency, we also compare the 1σ region (the probability of a burst in this region is \(\sim 0.68\)) of the two distributions in Fig. 3. It is found that the two distributions are most overlap. The results shown in Figs 2 and 3 agree well with that shown in Fig. 1.

5 DISCUSSION AND CONCLUSIONS

Based on the standard energy reservoir of GRB jets and the relationship between \(E_p\) and \(E_{iso}\), we test \(P(\theta)\) and \(P(\theta, z)\) predicted by the USJ model with observational-based simulations for a detection threshold of \(S > 4 \times 10^{-5}\) erg cm\(^{-2}\). We simulate a sample including \(10^6\) GRBs, and then derive \(P(\theta)\) and \(P(\theta, z)\). We find that both simulated and theoretical \(P(\theta)\) and \(P(\theta, z)\) are consistent with each other.

As can be seen in Figs 1–3, a small difference between simulated and theoretical \(P(\theta)\) and \(P(\theta, z)\) results from the range of \(\theta > 0.1\) rad. This difference increases as \(\theta\) increases when \(\theta > 0.1\) rad. The accumulative probability of \(\theta < 0.25\) rad derived from our simulations is \(\sim 99\) per cent, while it is 0.83 predicted by the USJ model. From equation (3) one can see that a burst with lower \(E_p\) tends to have a larger \(\theta\). Thus, we suspect that the difference might be due to the limitation of our simulations and the observational biases of \(E_p\) distribution. In our simulations, we derive \(\theta\) for each burst by equation (3), which requires \(E_p(1 + z) > 100\epsilon_{52}/k^{0.5} \sim 32(\epsilon_{51}/k)^{0.5} \sim 1\). Those bursts with \(E_p(1 + z) < 32\) keV are excluded from our simulated GRB sample. However, this limitation could not significantly affect our results as the \(E_p\) distribution used in our simulations mainly ranges in \(E_p > 100\) keV. The other possible reason might be the observational biases of the observed \(E_p\) distribution used in our simulations. For a burst with \(\theta > 0.25\) rad, its \(E_p\) value is less than 100 keV, assuming that \(\epsilon_{52} = 0.133, k = 1\) and \(z = 1\). Thus, the difference between the simulated and theoretical results at \(\theta > 0.25\) rad might be the bias of \(E_p\) distribution at \(E_p < 100\) keV. The distribution is taken from BATSE Large Area Detector (LAD) observations. The BATSE/LAD thresholds are sensitive to the spectral shape of a burst: the threshold for a hard burst is lower than the threshold for a softer burst.\(^2\) Bursts with low \(E_p\) might be more easily lost by BATSE/LAD. The BATSE/LAD operates in the 20–2000 keV band but usually triggers in the 50–300 keV band (e.g. Band 2003). BATSE generates a GRB record when the observed photon fluxes of two or three energy bands (normally 50–100 and 100–300 KeV bands) are simultaneously beyond the thresholds. In our simulations, we take a threshold as \(F_0 = S/T = 3.3 \times 10^{-6}\) erg cm\(^{-2}\) s\(^{-1}\), and then translate the \(F_0\) in units of \(\text{erg cm}^{-2}\ \text{s}^{-1}\) to photon cm\(^{-2}\) s\(^{-1}\) in a timescale of 1.024 s. We obtain 4.57 and 3.29 photon cm\(^{-2}\) s\(^{-1}\) in the 50–100 keV and 100–300 keV bands when \(E_p = 100\) keV, respectively. A normal GRB with such a photon flux is intense enough triggering the BATSE/LADs. However, the observed \(E_p\) distribution by BATSE/LADs is mainly in the range 100–1000 keV. This might suggest that BATSE/LAD sensitivity decreases significantly for \(E_p < 100\) keV (Band 2003), and bursts with \(E_p < 100\) keV are easily lost. If it is really the case, the \(E_p\) distribution in our threshold setting might still suffer a little of biases for \(E_p < 100\) keV. The upcoming \textit{Swift} satellite, which will be scheduled for launch in 2004 September (Gehrels 2004), is marginally more sensitive than BATSE for \(E_p > 100\) keV but significantly more sensitive for \(E_p < 100\) keV (Band 2003). It might establish a more reliable \(E_p\) distribution, which may be used for a further examination of our results.

The true GRB rate as a function of redshift is difficult to determine from the current GRB sample. We assume that the GRB rate is proportional to the star formation rate. Various models of star formation rates are presented in the literature, and they are quite different. In our analysis, we focus on the comparison of the simulated \(\theta\) distribution with that predicted by the USJ model. We find that the comparison result is not significantly affected by the redshift

\(^2\) http://www.batse.msfc.nasa.gov/batse/
Testing the predictions of USJ model of GRBs

Figure 4. Same as Fig. 1 but the SF2 model is used.

distribution. In our analysis, the SF1 model from Porciani & Madau (2001) is used. We use the SF2 model (Porciani & Madau 2001) to perform the simulations. The results are shown in Fig. 4. One can see that the comparison of the two distributions is not significantly affected by the model of redshift distribution.

Based on our results and the above discussion, we conclude that both the probability distributions, \( P(\theta) \) and \( P(\theta, z) \), derived from our simulations are consistent with that predicted by the USJ model. This consistency may be regarded as a support of USJ model.

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