Influence on simulation accuracy of atmospheric electric field around a building by space resolution


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1. Introduction

Although atmospheric electricity is composed of a wide range of electric phenomena in the troposphere, stratosphere, and even in the lower ionosphere (Qie, 2012), the influence of a complex underlying surface on atmospheric electric field is a key problem. The effects of tall buildings on atmospheric electric field intensification play an important role in forming corona layer at or near the ground (Aleksandrov et al., 2001, 2005a,b; Qie et al., 1994; Standler and Winn, 1979), initiating upward leader or lightning (Becerra and Cooray, 2006a,b; Jiang et al., 2013; López et al., 2013; Wang et al., 2012) and calculating lightning rod efficacy (D'Alessandro, 2003a,b; Ilčić and Aleksić, 2009; Moore, 1983; Moore et al., 2003). What's more, the intensification is caused by the instrument itself, so that the atmospheric electric field as measured by field mill needs to be corrected before it can be used (Bennett and Harrison, 2008; Minamoto and Kadokura, 2011; Qie et al., 2009; Serrano et al., 2006; Soula and Georgis, 2013; Xu et al., 2013). Thus, an accurate measurement or calculation of atmospheric electric field intensification, becomes a critical task for the scientific researcher who is interested in the problems mentioned above. Because of the limitation of the current field observation, the electric field around the buildings can hardly be measured effectively. However, with the rapid development of computer technology, the numerical calculation has been widely used to acquire the electric field around a building or a lightning rod. The purposes of the numerical calculations in existing literatures are mostly focused on the following two categories: 1) the magnitude of the electric field which is intensified by the building and 2) the relation between the electric field intensification and the building dimension.

The former category is mostly concerned researchers who study the effects on other atmospheric physical processes of the atmospheric electric field intensification caused by complex underline surface. It is important to calculate the electric field threshold value of corona ions releasing in corona discharge numerical simulations and air breakdown in upward leader initiating and propagating models (Aleksandrov et al., 2001,
What's more, the value of electric field intensification upon top of a building or a lightning protection rod is one of the determining factors to calculate the probability of lightning strike and the radius of protection of the lightning rod (Carrara and Thione, 1976; Moore et al., 2000; Petrov and Waters, 1995). Researches about the latter category mainly pay attention on the relationships between electric field intensification and a building's or a structure's dimensions. Hartmann (1984) discussed the relation of a tip's radius with the local electric field at the tip of a rod for seeking the self-sustained condition of stable corona discharge. D'Alessandro (2007, 2003a,b) showed the variations of electric field intensification caused by different dimensions of a building or a lightning rod, and different locations of rod placed on a building. Furthermore, the electric field intensified by different kinds of structures such as cylinder (Eriksson, 1979), ellipsoid (Moore, 1983) and so on, was also computed.

No matter whether the charge simulation method, the finite element method or the finite difference method is used, the value of the field intensification of a certain building or the relationship between the values and the dimensions was computed by dividing the space (in the following, we use continuous space which means that grid spacing is infinitesimal and approaches to zero, to express the space mentioned above in order to distinguish with discrete space) into discrete grids in numerical models. However, the calculations were different in different mesh spacing (D'Alessandro, 2003b; Tan et al., 2006; Tao et al., 2009). The uncertainty of the result caused by mesh spacing is unavoidable in numerical simulated works. And it will make a great negative impact as follows: 1) the universality of the calculation result and 2) the comparability of the results in different numerical studies. In choosing the correct threshold values of upward leader or lightning initiation and corona discharge on objects, the values would be different in numerical simulations with different grid spacing (Aleksandrov et al., 2006; D'Alessandro, 2003b; Lalande et al., 2002; Lalande and Mazur, 2012; Mazur et al., 2000). Thus, two important but hard problems have to be resolved: 1) how to reduce this uncertainty in numerical simulation and make the calculation results or some threshold values of different numerical simulations more comparable, and 2) whether the value of the electric field intensification can be computed or estimated in continuous space.

This paper aims to resolve these problems via various fine-spatial resolution calculations. Applying the FDM (finite difference method) to resolve Laplace's equation, we calculated the value of the field intensification which is produced by structures in different resolutions and then estimated the value in continuous space. Furthermore, according to the obtained values, the systematic errors in different resolutions were enumerated. The factors affecting the estimated value were also discussed in our paper.

2. Model and method

In this paper, the study is mostly focused on the method of estimating the value of electric field intensification factor on symmetric structure in continuous space and calculating the systematic errors in different resolutions. The space resolution adopted in simulation is an essential factor for calculation. The finer resolution is closer to the continuous space. Considering limiting of computer, the finer resolution can be adopted in 2D (two-dimension) model than 3D (three-dimension) model. So, the 2D Atmospheric Electric Field Intensification Model (defined as 2D-AEFIM) has been established. We focus on the electric field upon the cuboid structure's top corner where the intensification is most obvious. The area near the ground surface is considered as the main study area, which range is 400 m × 400 m. In addition, the background electric field (shown as E₀) is assumed as a homogeneous field, without the effect of free charge. The magnitude and the direction of E₀ is regarded as the same as the fair weather electric field near the surface of the Earth (Wallace et al., 2006), which magnitude is averaged ~130 V/m and direction is vertical downward. The atmospheric electric field distribution around the building has been calculated by using Laplace's equation. Five-point finite difference method is used to solve the Laplace's equation under the given boundary conditions in discretization field. Four boundaries of the model are divided into two categories: one bottom boundary, which is formed an equipotential surface of 0 V and composed of earth and well-grounded structure, follows Dirichlet boundary condition; the other three are air boundaries, including two lateral and one top boundaries, all follow Numann boundary condition. And the parameters of the structures, which affect the electric field intensification, are considered as height (H) and width (W).

The grid spacing (delegated as h) in the X and Y directions has been adopt to be equal and 10 different values are set in our simulations. So there are 10 different resolutions for each building pattern to estimate the value of electric field intensity in continuous space. It needs to be declared that the finer resolution is associated with the smaller h and the larger h delegate the coarser resolution in a fixed simulation region.

With a structure present, Laplace’s equation, \( \nabla^2 \phi = 0 \), is solved by using the FDM. Since the number of grid is larger, we use SOR (successive over-relaxation) iterative algorithm to solve difference equation to acquire the potential (Mansell et al., 2002). The iterative formula of potential is as follows:

\[
\varphi_{i,j}^{n+1} = \varphi_{i,j}^{n} + \omega \left( \varphi_{i-1,j}^{n} + \varphi_{i+1,j}^{n} + \varphi_{i,j-1}^{n} + \varphi_{i,j+1}^{n} - 4\varphi_{i,j}^{n} \right) / 4 \tag{1}
\]

where \( \varphi \) is electric potential, and \( \omega \) is over-relaxation parameter which has an experimentally determined value in the range of 1 to 2.

This solution provides potential of each grid point over the problem region, shown in Fig. 1. The magnitude of the electric field intensity is computed from potential gradient, \( E = -\nabla \varphi \) (D'Alessandro, 2007).

The point of interest on a structure is the nearest point upon the corner of the top plant surface. The electric field intensification factor \( K_i \) of that point means the ratio between the magnitude of \( E \) of the point and background electric field intensity \( E_0 \). In this paper, we neglected the impact of corona layer on the electric field intensity over the corner or the tip when the value of the intensified field intensity is greater than the threshold value of corona ion emission. And just focus on the effect of electric field intensification itself. The main variables in our study are as follows: structure height, \( H \); width, \( W \); and grid spacing, \( h \). Multiple non-linear regression fits are carried out on the \( K_i \) data to obtain general relations. Given the above information, it can be seen that \( K_i = f(h,H,W) \).
The variables and constants mentioned in this section are summarized and illustrated as follows:

The variables that describe the physical dimension of a structure are:

\[ H \] height of a structure, contained 6 different values, respectively are 10 m, 20 m, 40 m, 60 m, 80 m, and 100 m; and

\[ W \] width of a structure, included 6 different values, respectively are 2 m, 4 m, 6 m, 8 m, 10 m, and 20 m.

So there are 36 building patterns with different dimensions in our simulation.

The variable that describes the spatial resolution of the model is:

\[ h \] the grid spacing of the model, and the values are set as follows: 1 m, 0.5 m, 0.25 m, 0.2 m, 0.125 m, 0.1 m, 0.05 m, 0.04 m, 0.025 m, and 0.02 m, 0.10 different resolutions for each building pattern.

The variables and constants that describe the atmospheric electric field are:

\[ E_0 \] ambient atmospheric electric field, magnitude is 130 V/m and direction is vertical downward; and

\[ E \] intensified atmospheric electric field of the interest point.

The variable that describes the value of the atmospheric electric field intensification of the interest point is:

\[ K_i \] electric field intensification factor which equals to the magnitude of \( E/E_0 \).

**3. Data and data analysis**

**3.1. Characteristics of atmospheric electric field distortion near a structure in continuous space**

In common sense, continuous space means that \( h \) is infinitesimal and approaches to 0. Since smaller \( h \) means much more grids in the same simulation area, SOR iterative algorithm adopted for electric field calculation costs more elapsed time with more grids. Most importantly, the resolution adopted by numerical simulation is limited obviously, considering computer’s internal storage. Therefore, it is unattainable that \( h \) approaches to or equals to 0 in numerical simulation. However, the finer resolution means that the calculated result is closer to the fact. It is a key how to improve the computing accuracy, under current computer condition.

Accordingly, other method should be considered on calculating or estimating the value of electric field intensity around a structure in continuous space. Taking the closest point upon the top corner as research object, the values of \( K_i(h,H,W) \) with different resolutions are different (shown in Fig. 2(a)), in this example, \( H = 20 \text{ m}, W = 8 \text{ m} \); \( K_i(h,H,W) \) can be described as \( K_i(h,20,8) \), which are decreasing with the increasing \( h \). The relationship between \( K_i(h,20,8) \) and \( h \) is exponential due to fitted curve. The square of correlation coefficient (shown as \( R^2 \)) is equal to 0.99 at the confidence level for the curve of 95%, and the fitted formula is as follows:

\[
K_i(h, 20, 8) = 125.535 \exp(-h/0.02954) + 8.0757. \tag{2}
\]

According to the limit of the above fitted formula when \( h \) approaches to 0, \( K_i(0,20,8) \) can reach to 134.6107. Although the value estimated in our calculation is greater than the atmospheric electric field intensification factor ~50 computed in discrete space in finite element method for the point on the corner of a rectangular structure of width 30 m and height 20 m (D’Alessandro, 2003b), but our simulated result is consider to be reasonable with the published result, by reason that the width of a structure is more narrow and the grid spacing is so fine that approach to zero in our research. The \( K_i(0,20,8) \) can be regarded as the value in the continuous space, estimated from the function (Eq. (2)) which is built based on other ten \( K_i(h,20,8) \) calculated in 10 different resolutions. It can also be regarded as the standard value. It needs to be known that the standard value mentioned in our paper doesn’t represent the unique value in nature. However, this extrapolated value might be regarded as a data standard for error analysis of simulation works within discrete grids. In order to distinguish the extrapolated value from the numerical calculated value \( K_i(h,H,W) \), we use \( K_i(H,W) \) to replace the \( K_i(0,H,W) \). Thus, this exploration method is expressed as the following Eq. (3):

\[
K_i(H, W) = K_i(0, H, W) = \lim_{h \to 0} \{K_i(h, H, W)\}. \tag{3}
\]

The \( K_i(H,W) \) of other structures (35 patterns in total) can be obtained by the same extrapolation method. Based on estimated values, \( K_i(H,W) \) is linear with \( H \) when \( W \) is fixed (Fig. 2(b), as an example, \( W = 2 \text{ m} \)), and exponential with \( W \) when \( H \) is fixed (Fig. 2(c), as an example, \( H = 100 \text{ m} \)).
The fitted formulas are shown in Eqs. (4) and (5), and each $R^2$ is greater than 0.989 at the confidence level for the curve of 95%.

\begin{equation}
K_r(H, 2) = 4.87H
\end{equation}

and,

\begin{equation}
K_r(100, W) = 94.394 \exp(-W/1.1252) + 442.67.
\end{equation}

As shown in Fig. 1, the $K_r(H, W)$ is associated with not only the height but also the width of a structure. The forms of the atmospheric electric field intensification in continuous spacing variation with a structure’s height and width are the same with that in discrete space investigated in the publications (D’Alessandro, 2003b). It is not clashed with the most observations and calculations (D’Alessandro, 2007; Eriksson, 1987; McCann, 1944; Pierce, 1971; Rakov and Uman, 2003; Zhang et al., 2009) that the electric field intensification over higher structure is greater than shorter one and hence the higher probability that a stepped leader will terminate on the higher objects. And also not go against with the laboratory measurements that the electric field intensification is much stronger over sharp-tipped electrodes than over blunter ones before to any emissions (Moore et al., 2000), but the corona ions which act to reduce the local field are also easy formed over the sharp one.

In order to acquire a fitted relationship formula between $K_r(H, W)$ and a structure dimension, multivariate non-linear regression method has been chosen, following the existed work (D’Alessandro, 2003a). Different from a certain resolution adopted in this work, a universal relation is presented as Formula (6), based on the estimated $K_r(H, W)$ of all building patterns in continuous space. Formula (6) passed homogeneity of variance test.

\begin{equation}
K_r(H, W) = 14.5797H^{0.7478}W^{-0.0124}.
\end{equation}

It can be found from Eq. (6) that the value of $K_r(H, W)$ is mainly contributed by structure height. It is albeit mildly that the $K_r(H, W)$’s variation with the width, but any analysis of practical structure must consider these differences. Eq. (6) can

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**Fig. 2.** Atmospheric electric field distortion near the tip in continuous space. (a) $K_r$ in continuous space is estimated by the fitted formula (dashed line is the fitted curve) which is based on the given computed values of different resolutions represented as 10 solid points. The structure’s width is 8 m and height is 20 m. (b) Using extrapolation method as (a), $K_r$ of different structures can be obtained. The linear relationship between $K_r$ and $H$ in continuous space is shown. Here, taken as $W = 2$ m as an example. Solid points are estimated results and dashed line is fitted curve. (c) is the same as (b) but which indicated the relation of $K_r$ and $W$, when $H$ is fixed at 100 m.
be used to analyze the atmospheric electric field intensification at the point near the top corner of a rectangular or cylindrical structure with any dimension in continuous space. Then, the error of the calculated value in a certain resolution can be easily acquired when the extrapolated value is certain. And the error analysis has been presented in the next section.

3.2. Systematic error analysis of different resolution

Comparing the difference between the calculated value of \( K_r(h,H,W) \) in a certain resolution and the extrapolated value \( K_r(h,H,W) \) in continuous spacing, the systematic error (shown as \( \eta(h,H,W) \)) is obtained. And \( \eta(h,H,W) \) of different structures in different mesh spacing are counted by Eq. (7):

\[
\eta(h,H,W) = \frac{|K_r(h,H,W) - K_r(h,H,W)| \times 100\%}{K_r(h,H,W)}
\] (7)

\( K_r(h,H,W) \) the estimated value of field distortion in continuous spacing, when \( h \) approaches to 0, the detailed calculating method has been shown in 3.1; \( K_r(h,H,W) \) the field distortion value in a certain mesh spacing.

According to the above calculation, the numbers of systematic error \( \eta(h,H,W) \) are 360 in total associated with 36 structure patterns and 10 mesh spacing. The \( \eta(h,H,W) \)'s mean value of 36 structure patterns at each mesh spacing is given in Table 1. And in Table 1, SD means the standard deviation which is mainly caused by the differences of the dimensions. As shown in Table 1, the absolute values of SD all are less than 1%, which indicates that \( \eta(h,H,W) \) is a parameter co-related merely with mesh spacing rather than structure dimension. Therefore \( \eta(h,H,W) \) can be simplified into \( \eta(h) \). Based on Formula (7), the range of \( \eta(h) \) must be 0–100% because the \( K_r(h,H,W) \) has a value between 0 and \( K_r(h,H,W) \). Therefore, the function of the form of logistic can be chosen for linear fitted about \( \eta(h) \). The logistic correlation between \( \eta(h) \) and \( h \) is shown in Eq. (8) and the \( R^2 \) is 0.975 at the confidence level for the curve of 95%.

\[
\eta(h) = \left\{1 - 1/\left[1 + (h/0.024)^{1/2}\right]\right\} \times 100\%.
\] (8)

3.3. Discussion of the exploration method

As indicated in Table 1 and Fig. 3, the error is growing significantly with enlargement of mesh spacing. Large mesh spacing is often chosen to reduce time-cost, in practical application, but it will accompany with the large systematical error. Eq. (9) is given for transformation from a computing value to the extrapolated value in continuous space.

\[
K_r(h,H,W) = K_r(h,H,W)/[1 - \eta(h)]
\] (9)

where \( K_r(h,H,W) \) means the field distortion value in a certain mesh spacing is from numerical modeling and \( \eta(h) \) can be estimated according to Eq. (8).

Despite of structure dimensions, the equation about systematic error and grid spacing can be used to acquire the electric field intensification as a fine resolution calculated result but just adopting coarse resolution. Furthermore, the conclusion contributes to the comparison of results in different resolution model works on electric field intensification, especially when it comes to magnitude of electric threshold (such as initiation electric threshold of upward leader, upward lightning and corona discharge originating from a structure tip) in different resolution simulations.

### Table 1

<table>
<thead>
<tr>
<th>Mesh spacing ( h ) (m)</th>
<th>Systematic error ( \eta^a ) (%)</th>
<th>SD(^b) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>96.31 ± 1</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>95.12 ± 1</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>93.03 ± 1</td>
<td></td>
</tr>
<tr>
<td>0.125</td>
<td>92.01 ± 1</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>88.92 ± 0.5</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>86.85 ± 0.5</td>
<td></td>
</tr>
<tr>
<td>0.04</td>
<td>76.27 ± 0.5</td>
<td></td>
</tr>
<tr>
<td>0.025</td>
<td>70.91 ± 0.5</td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>54.84 ± 0.5</td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>44.12 ± 0.5</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) Systematic error mean value.
\(^b\) Standard deviation.
methods are adopted for the fitted formula to estimate $K_r$ of the same structure (height of 20 m, width of 8 m). As shown in Fig. 4, all points in box A and B are the estimated $K_r(20,8)$, and the points of $K_r(20,8)$ on line 1, 2, and 4 are in A while others on line 3 and 5 are in B. It can be found that all of the $K_r(20,8)$ in box A are extrapolated from the lines containing smallest $h$ of 0.02 m, different from the $K_r(20,8)$ without $h$ of 0.02. The conclusion can be drawn that the estimated extrapolated value has a significant correlation with the smallest mesh spacing chosen for the fitted formula. Inversely, the effects of the fitted points' number and distribution are less on the extrapolated value.

4. Conclusion

Based on amount of numerical simulation results, the extrapolation method is used in this paper to estimate the extrapolated value of the atmospheric electric field intensification factor ($K_r(H,W)$) about the point near a structure's top corner in continuous spacing, which overcomes the difficult problem in numerical computation. Comparing the values in continuous space and different resolutions, the systematic error of a certain resolution has been calculated. And the main conclusions are as follows:

1) In continuous spacing, the relationships between the $K_r(H,W)$ and a structure’s dimension parameters are revealed. The $K_r(H,W)$ is linear increasing with the growth a structure's height and exponential decreasing with the width widening. And the non-linear regressive equation of $K_r(H,W)$ is established.

2) The systematic error that caused by a certain resolution in numerical simulation is a fixed value, which is associated with the resolution rather than a structure’s dimension.

3) The estimated extrapolated value has a significant correlation with the smallest mesh spacing chosen for the fitted formula, but the effects of the fitted points' number and distribution are less on the extrapolated value.

5. Discussion

Because of the limitation of the computer conditions, we only use two-dimensional model to discuss the intensification. It makes a structure or the tips form rectangular mostly. And the estimated extrapolated value may have some errors because the extrapolation method is based on the fitted formula. However, the uncertainty caused by grid spacing is inevitable and it should be taken into consideration in numerical calculations. The simulated results are considered to be beneficial not only to improve the calculation accuracy of electric field intensifications in simulation, but also to compare the results of different simulation works in different resolutions. In addition, it might be a practical way to decide value of frequently-used electric thresholds when the grid spacing are different, such as the initiation threshold of upward leader, upward lightning initiation and corona discharge originating on a tall structures or tips.

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