Equivalent-nanocircuit-theory-based design to infrared broad band-stop filters

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Abstract: We theoretically introduced a design paradigm and tool by extending the circuit functionalities from radio frequency to near infrared domain, and a broad band-stop filter, is successfully demonstrated by cascading triple layers of nano-square arrays. The feasibility is confirmed by its consistency with the rigorous FDTD calculation. Moreover, such a third-order Butterworth filter is not only insensitive to the incident angle but also to input light’s polarization. The new paradigm forms a theoretical foundation for designing optical devices and also enriches the classic circuit operations at the optical frequency region.

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References and links
1. Introduction

Frequency selective surfaces (FSSs) have been widely investigated for their widespread applications as filters for decades. Using circuit elements, [e.g., resistors (R), inductors (L), and capacitors (C)], FSSs can be effectively and flexibly designed in the microwave and THz domains [1–3]. There has also been a great interest in pushing FSS’s application in higher frequency range to achieve high-density, and high-speed optical analogues [4–6]. Generally, by just simply reducing the sizes of the basic units into micrometer and nanometer level, it is technically difficult to achieve this goal. In this direction, Engheta et al. have made an important breakthrough by using optical nanoparticles instead and pointed out that nanoparticles, when properly designed and judiciously arranged, can behave as nanoscale lumped circuit elements for an optical field [7]. This is consistent with the dispersion properties of plasmonic materials [8] and surface plasmon polaritons at optical frequencies. A collection of such nanoparticles may then form a “circuit of light”, which, when excited by an optical signal, shows local electric fields and displacement currents analogous to voltage and current distributions in conventional radiofrequency circuit [9, 10]. This new concept of metamaterial-inspired optical nano-circuitry, dubbed “metatronics”, has gained various successful applications which are consolidated by the consistency between a series of theoretical analyses and experimental realizations [11–14].

For example, Engheta et al. presented a designable near-infrared (NIR) lumped nanocircuit with tailorable response based on a simple nano-rod geometry, and constructed band-pass or band-stop filters by evaluating their equivalent impedance as lumped circuit elements [15]. However, such a structure is polarization-dependent, and moreover the response band is relatively narrow, which are both disadvantageous to various applications. This inspires us to find a more suitable optical structure to avoid these points which maybe rely on more flexible or simple designing strategy.

Here, a synthesis procedure for designing a band-stop optical FSS in infrared optical region is proposed by combining the design methods of FSS in microwave or THz domain with those lumped nano-circuit ideas. Periodic arrays of square particles with subwavelength scales are utilized as the synthesis blocks. First, a single layer of such a periodic nano-square array (NSA) is analyzed and demonstrated to have the band-stop property with a suitable choice of size and period parameters. Then, by cascading triple layers with suitable sizes, a third-order Butterworth band-stop filter is realized. Such a filter is possessing with broadband, wide angle-insensitive and polarization-independent properties, which are confirmed by full-wave finite difference time domain (FDTD) simulations and the equivalent-circuit (EC) theory.
2. Methods

The general configuration of one layer of periodic nano-square array used for FSS filter is presented in Fig. 1(a). The width and height of its unit cell, as well as the separation air gap between the two consecutive unit cells are denoted respectively by $w$, $h$ and $g$. The material for making up of the cells is indium-tin-oxide (ITO), which is widely used as an electrode for displays because of its low electrical resistance and high transmittance in the visible range [16]. Now it is clear that the fundamental difference between a conventional circuit and one operating in the optical regime is, at optical frequencies one has to worry about not only capacitance, resistance, and conventional inductance, but also the kinetic inductance of the conduction electrons in the metals, the ITO here, or other materials [17].

When such a nano-square array is illuminated by an optical signal from the bottom side vertically (along $z$ direction) with electric field $E$ polarized parallel to the $x$ direction, the optical displacement current $\partial D/\partial t$ “flows” along both the unit cells and air gaps [7], and here time-harmonic behavior with time convention $e^{-i\omega t}$ is assumed. The nano-square array first acts as a “parallel” combination of lumped elements $R$, $L$ and $C_1$, and then in series with a capacitance $C_2$, as shown in Fig. 1(b). In addition, because at optical frequencies, the real of ITO material’s permittivity $Re(\varepsilon_{ITO}) < 0$ and the imaginary part $Im(\varepsilon_{ITO}) \neq 0$, the unit cell would act as a parallel combination of a nano-inductor $L$ and a nano-resistor $R$ (Fig. 1(b)).

Following the general capacitor impedance formula $Z_c = i/(\omega C)$, with the capacitance $C = \varepsilon a / b$, and $a$, $b$ being the element’s two dimensions, the effective lumped impedance of the unit nano-square cell in Fig. 1 can be written as $Z_{ns} = iw / (\omega h e_{ITO})$. Similarly, due to the permittivity of air gap is $\varepsilon_{air} = 1$, larger than zero, therefore, the parallel capacitive impedance of the air gap with nano-square cell can be written as $Z_{c1} = i g / (\omega h e_{air})$, and the serial capacitive impedance of the air gap with nano-square cell is given as $Z_{c2} = ig / (\omega h (w + g)e_{air})$.

Above all, the equivalent circuit model of this whole FSS system is shown in Fig. 1(c), where $\eta = \sqrt{\mu / \varepsilon}$ is the intrinsic impedance of surrounding medium. Within the equivalent circuit impedance theory [18], the light reflection coefficient from the nano-square array is derived as: $Z = Z_{ns} || Z_{c1} + Z_{c2}$, $S_{11} = (Z\eta - \eta) / (Z\eta + \eta)$, where “$||$” is the parallel symbol.
The transmission coefficient can be obtained as \( S_{21} = S_{11} + 1 \), which is exactly valid only if the thickness of the array is negligible. Then, the reflectance and transmittance of the incident optical signal are naturally obtained,

\[
R = |S_{11}|^2, \\
T = |S_{21}|^2 = \frac{2Z}{(2Z + \eta)^2}
\]

Obviously, the reflection and transmission properties of this FSS filter are dependent on the size of the structure (that is \( w, h \), or \( g \)), the constituent materials properties (\( \varepsilon_{\text{ITO}} \)), or the wavelength of the illumination source (\( \lambda = \frac{2\pi c}{\omega} \)).

3. Results and discussion

In the following calculations, the material permittivity of ITO is modeled using the Drude dispersion:

\[
\varepsilon_{\text{ITO}}(\omega) = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega^2 + i\omega\tau} \quad \text{with} \quad \varepsilon_{\infty} = 3.91, \quad \omega_p = 2.65 \times 10^{15} \text{ rad/s} \quad \text{and} \quad \omega_\tau = 2.05 \times 10^{14} \text{ rad/s}.
\]

If the nano-square cell’s width (\( w \)) or separation gap (\( g \)) in order to achieve different optical responses, the impedance of the nano-circuit element can be changed, and then the property of such a band-stop filter could be flexibly controlled. Here, we explore four examples from A to D aiming for an optimization to the filtering effect and the sample’s geometric parameters are listed in Table 1.

### Table 1. Geometrical parameters of the samples.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Width (( w ))</th>
<th>Gap (( g ))</th>
<th>( w/g )</th>
<th>Height (( h ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100 nm</td>
<td>100 nm</td>
<td>1</td>
<td>150 nm</td>
</tr>
<tr>
<td>B</td>
<td>200 nm</td>
<td>100 nm</td>
<td>2</td>
<td>150 nm</td>
</tr>
<tr>
<td>C</td>
<td>300 nm</td>
<td>100 nm</td>
<td>3</td>
<td>150 nm</td>
</tr>
<tr>
<td>D</td>
<td>400 nm</td>
<td>100 nm</td>
<td>4</td>
<td>150 nm</td>
</tr>
</tbody>
</table>

![Fig. 2. Transmittance spectra obtained from (a) the theoretical calculation based on Equivalent circuit theory and (b) numerical calculation based on FDTD algorithm for four samples (A to D). (c) The variance of the resonance wavelength (band-stop center) and transmittance dip (band-stop depth) versus the ratio w/g for theoretical (squared) and FDTD numerical calculations (circled). (d) The corresponding electric field distribution for sample B at resonance wavelength 2097nm is also shown.](image-url)
The transmittances of the FSS filters from these samples, shown in Fig. 2(a), are obtained via the Eqs. (1) and (2) in Section 2. The band-stop center red-shifts and depth increases when changing sample from A to D, that is, increasing the unit cell width from 100nm to 400nm. To check the validity of such an equivalent nano-circuit theoretical model and also make it clear why these response changing behavior quantitatively, we employ a full-wave EM simulation by using three-dimensional (3D) FDTD algorithm [19, 20], where nano-square array is modeled with periodic boundary conditions in the transverse x-y plane and the perfect match layer (PML) boundary condition along the propagation z direction. In order to get accurate results, the mesh size is set as 5nm in all directions, and the simulation accuracy is guaranteed as convergent via the reduction of the mesh size in three directions by half. As shown in Fig. 2(b), the corresponding transmittance spectra simulated by FDTD methods are consistent well with those theoretical results in Fig. 2(a).

The variation of the band-stop center wavelength (corresponding to the resonant wavelength) and depth (corresponding to the transmittance dip) as a function of w/g is investigated, and the corresponding results are shown in Fig. 2(c). With the increment of w/g, the band-stop center has a red-shift and the band-stop depth decreases. The red-shift behavior can be well interpreted by the optical circuit theory [21]: The larger width of nano-square unit cell, the bigger inductor L induced between two adjacent air gaps, hence leading to a higher resonant wavelength. On the other hand, as seen from the electric field distribution for sample B is shown in Fig. 2(d) at its resonant wavelength of 2097nm. The nano-square unit cell, working as an antenna, localizes the incident light, resulting in a significant resonant enhancement of localized field and a guidance of most light through the air gap [22, 23]. The larger width of nano-square unit cell is, the stronger of the guidance of light, which is the reason for the depth’s increment.

As for a more practical and wide application, a flatter and broader band-stop filtering response curve with a fast roll-off would be much advantageous [24]. To gain this aim, as the general FSS-based filter design scheme does [25–27], a third-order Butterworth band-stop filter is realized by cascading triple-layer of nano-square unit cells with a specific separation distance D between the consecutive layers (Fig. 3(a)). Here D is set as one quarter of the central wavelength of the incident light. This is based on the consideration that because quarter-wavelength sections of line between the layers act as admittance inverters to...
effectively convert alternate shunt resonators to series resonators, the transmission line sections are $\lambda_0 / 4$ long at the center wavelength $\lambda_0$ in order to convert the triple layers into series of branches [25]. Therefore, the corresponding equivalent circuit of this third-order FSS system can be modeled as that shown in Fig. 3(b). Then, the whole structure is separated into 4 regions along the propagation direction (Fig. 3(c)), corresponding to the three nano-square array layers one by one. Finally, along the similar procedure, the transmittance $T$ after the third layer can be obtained step by step: 1) for the first layer, the theoretical $S$-parameters are:

$$S_{11} = \frac{Z_i - \eta}{Z_i + \eta}, \quad S_{21} = S_{11} + 1, \quad S_{22} = S_{11}, \quad S_{12} = S_{21}. \quad (3)$$

Here, the impedance of the surrounding medium above or between the neighbored layers is $\eta$ and that of each layer is $Z_i$ ($i = 1, 2, 3$). As for the second or third layer, the $S$-parameters ($S_{13}, S_{43}, \ldots$) can be get by just replacing $Z_1$ in formula (3) to $Z_2$ or $Z_3$. 2) The reflected power $P_{1R}$ and transmitted power $P_{1F}$ from each layer are evaluated as follows:

$$P_{1R} = |S_{11}|^2 P_{1F} + |S_{12}|^2 P_{2R}, \quad P_{2R} = |S_{21}|^2 P_{1F} + |S_{22}|^2 P_{2R}, \quad P_{3R} = |S_{31}|^2 P_{2F} + |S_{32}|^2 P_{3R}, \quad P_{3F} = |S_{41}|^2 P_{2F} + |S_{42}|^2 P_{3R} \quad (4)$$

$$P_{4R} = |S_{51}|^2 P_{3F}, \quad P_{4F} = |S_{65}|^2 P_{3F}.$$

For simplicity, the absorption loss by the surrounding medium negligible is assumed. 3) The total transmittance after the third layer is then written as

$$T = P_{4F} / P_{1F}$$

$$= \frac{|S_{21}|^2 |S_{43}|^2 |S_{65}|^2}{1 - |S_{11}|^2 |S_{32}|^2 - |S_{52}|^2 |S_{65}|^2 + |S_{11}|^2 |S_{33}|^2 |S_{65}|^2 - |S_{11}|^2 |S_{43}|^2 |S_{55}|^2}. \quad (5)$$

Using parameters in Table 2 and the above formula (5) from equivalent circuit theory together with the FDTD algorithm, the theoretical and numerical transmittance spectra for this new triple-layer (3-order) filter are shown in Fig. 4 (a). The corresponding results for the case that only the bottom (first) layer is remained are also shown for a direct comparison. Triple layers replacing single layer not only makes the band-stop’s width (1.5 $\mu$m–3 $\mu$m), and depth become larger, but also the whole band-stop bottom be much flatter. More importantly, the transmittance is totally zero, which is also clearly indicated in the corresponding electric field distribution in Fig. 4(b) (where 2000nm wavelength is chosen for example). After the third layer, there is no light left. The corresponding FDTD calculation results indicate a good agreement with the equivalent circuit theoretical calculation, for example the band-stop center or depth. The slight deviation at the filter bottom width could arise from the simple assumption that the triple nano-square array layers are independent just for the convenience of calculation. However anyway, the proposed synthesis procedure is confirmed as qualitatively feasible to provide us a way to design a third-order Butterworth band-stop filter. This procedure can be naturally and easily extended to design other nano-optical devices.
Fig. 4. (a) The transmittance spectra from the equivalent circuit theory for single-layer (1-order) filter (dashed dotted line) and triple-layer (3-order) filter (dashed line). The full-wave FDTD simulation results (dotted and solid lined) are also shown for comparison. (b) The electrical field distribution with 2000nm wavelength being chosen for example in the case of 3-order filter.

Table 2. The element values for the low-pass prototype circuit, and the geometrical parameters of the third-order band-stop FSS filter for the design example.

<table>
<thead>
<tr>
<th>i</th>
<th>G_i</th>
<th>Z_i</th>
<th>w/g (nm)</th>
<th>h_i (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Z</td>
<td>120/60</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>Z/2</td>
<td>120/60</td>
<td>300</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>Z</td>
<td>120/60</td>
<td>150</td>
</tr>
</tbody>
</table>

The equivalent-circuit-theory-based design procedure, outlined above, introduces a theoretical foundation. As for any specific third-order Butterworth filter with such as a desired central band-stop frequency and a desired band-stop width, one can easily get the suitable choice of the geometrical parameters for design. For example, if the band-stop edge frequencies are \( \omega_1 \) and \( \omega_2 \), the center frequency can be defined as \( \omega_c = \sqrt{\omega_1 \omega_2} \) and the band-stop width as \( \omega_b = (\omega_2 - \omega_1) / \omega_c \). The reactance slope parameters of this circuit model in terms of low-pass prototype parameters \( G_0, G_1, \ldots, G_n+1 \) and cut-off frequency \( \omega_c \) are given as \cite{18}:

\[
\frac{Z_i}{Z_0} = \frac{G_i}{\omega_c G_i \omega_b} \quad n = \text{even}, \quad \frac{Z_i}{Z_0} = \frac{1}{\omega_c G_i G \omega_b} \quad n = \text{odd},
\]

where \( G_i \) is the normalized prototype element values, and \( n \) is the order of the Butterworth filter, that is the number of the resonators. Since we consider the case where the whole structure is embedded in the same medium [e.g. air] implying \( Z_0 = \eta \), we design for \( n = \text{odd} \).

As for a third-order band-stop filter (\( G_1 = 1, G_2 = 2, G_3 = 1 \)) with the heights of nano-square cells at each layer respectively \( h_1 = 1/2 h_2 = h_3 \) fixed, the impedance of each layer (Fig. 3(b)) can be obtained as: \( Z_1 = 2Z_z = Z_3 = \eta / (\omega_c \omega_b) \). However, from Section 2, we know that the impedance for a single layer of nano-square array is \( Z = Z_{NS} \parallel Z_{c1} + Z_{c2} \) and the filter response is obtained from \( T = |S_{21}| = |2Z / (2Z + \eta)|^2 \). Then, by solving the equations \( Z = Z_i \) and \( \frac{\partial T}{\partial \omega_b} \bigg|_{\omega_c=\omega_b} = 0 \) simultaneously, the geometrical parameters (such as width \( w \) and gap \( g \)) can be calculated.

As for the real fabrication of this cascaded triple layers structure, the air gap is generally filled with some dielectric materials. For instance, SiO\(_2\) is taken as the host media, and further
investigation to the filtering property of this kind of third-order filter illuminated by oblique incidence is made. The FDTD numerical results are presented in Fig. 5 for a broad non-normal incident angles of TM or TE light from $\theta = 0^\circ$ to $50^\circ$ in the step of 10 degree. The frequency response of the FSS filter is stable. What’s more, due to its four-axis symmetrical property, the third-order band-stop FSS filter is also insensitive to the incident light’s polarization, no matter TE or TM wave.

![Fig. 5. Transmittance spectra of the designed 3-order optical FSS filter as a function of the incident angle for TM (a) and TE (b) waves, red line is the equivalent circuit (EC) result for the case of 0 degree incidence. Here SiO$_2$ are used as the surrounding material and the geometrical parameters are same as those at Table 2.](image)

4. Conclusions

In conclusion, the design procedure to a triple layer optical FSS filter, possessing the broadband, wide angle insensitive and polarization-independent properties, was successfully demonstrated, based on the equivalent nano-circuit theory. After extracting the equivalent impedance of each nano-square array layer, equivalent nano-scale circuit elements are modeled. The model’s validity is confirmed by its consistency with the FDTD calculation. Via cascading triple nano-square array layers, and considering the sensitive-dependence of band-stop property on the nano-square cell’s geometrical parameters, a third-order Butterworth filter with stop-band bottom being much broader, flatter, and transmittance being completely zero, was realized. Moreover, such a band-stop filter possesses the angle-insensitive and polarization-independent properties. Above all, this proposed design approach, based on equivalent circuit theory, provides us an efficient, flexible and powerful tool to design the optical meta-devices, filters, stealth system in visible and infrared region, and so on.

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