Modeling and testing of static pressure within an optical fiber cable spool using distributed fiber Bragg gratings

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\section{Introduction}

Since the fiber cable is impervious to conventional electronic counter-measures, it emits no exploitable radio frequency signals, provides extremely wide bandwidth and a high data transmission rate, it offers a major benefit for the fiber optic guided missile (FOG-M) technology [1]. In the FOG-M system, the fiber cable is firstly wound on a spool, then stored for later use, and finally released from the spool to serve as bidirectional communications media for transmitting both the situational data from the missile to the launch platform and the command guidance signals from the launch platform to the missile. Obviously, the winding and releasing techniques of the fiber cable are two key technologies in designing a reliable FOG-M system. Note that the pressure in the spool affects the fiber birefringence and thus the signal transmission quality, therefore the inner pressure analysis of the FOG-M fiber cable spool is not only necessary for designing the fiber cable winding and releasing mechanism, but also significant for analyzing the loss of the fiber cable and for predicting the lifetime of the fiber cable spool.

Since the fiber Bragg grating (FBG) has lots of advantages such as its small size, anti-interference ability, high sensitivity, good stability, long lifetime, wide monitoring range, real-time online measurement, easy to form smart sensor network and other advantages [2–6], it is widely used in sensing field [7–10]. FBG acts as an optical filter that reflects the incident light at a particular wavelength (Bragg wavelength), and this reflection wavelength changes in proportion to the applied strain or temperature change [11,12].

In this paper, we propose a new method to study the inner pressure distribution in the fiber cable spool. By introducing the distributed FBGs to the spool, we realize a real-time measurement to the fiber cable layer pressure by use of the FBG sensing characteristic of the radial direction strain. A theoretical model of the fiber cable layer's pressure distribution for the fiber cable spool was established. Theoretical simulations agree qualitatively with experimental results. Using this method, the problem of the fiber cable layer pressure real-time monitor during the cable winding process was solved. This research provides a good reference to the fiber cable winding, the fiber cable layer pressure measurement, the spool design, the fiber cable loss analysis and the lifetime prediction of the fiber cable spool in the FOG-M system. Moreover, this study might provide a valuable reference to the fiber winding in the fiber optic gyroscope [13] and to the filament winding in the composite pressure vessels [14], etc.
we can simplify the complicated winding device to a simple mechanism model, as shown in Fig. 1, for the convenience to the theoretical analysis. A constant tension force \( T \) to the fiber is provided by the weight \( G \). The fiber is wound to the spool layer by layer as the spool is accurately rotated and moved up and down. To establish a theoretical model of the fiber cable layer's pressure, we depict Fig. 2 to analyze the layer's force. In what follows, we gradually analyze this theoretical model from the simple condition to the close actual situation. Firstly, we analyze the simplest case where both the radial shrink of the fiber cable spool and the friction force between fiber cable layers are neglected. In this case, we suppose that the fiber cables align regularly along the spool axial direction and have no deformation. Secondly, we consider the case where the radial shrink of the fiber cable spool is involved. As in the actual spool of the FOG-M system, the pressure from the outer cable layers causes a slight displacement to the inner fiber cables, which generally makes the cables become tighter and experience a radial shrink. Finally, the friction force between fiber cable layers is further considered. Through these three steps, the ideal theory model is revised gradually and becomes suitable to the actual fiber cable spool of FOG-M.

2.1. Simplest case

Fig. 2 illustrates the force analysis of the fiber cable spool supposing that only the first fiber cable layer is wound on it. \( R \) is the spool radius, \( O \) is axis center of the spool, \( dl \) is the fiber cable length micro-element, \( d\theta \) is the plane angle corresponding to \( dl \), and \( N \) is the crutch force. Thus, the stress to the spool surface can be written as

\[
F_0 = -N = 2T \sin \frac{d\theta}{2}. \tag{1}
\]

Consider that the relation of \( \sin(d\theta/2) \approx d\theta/2 = dl/2R \) is always satisfied since \( dl \) is very small for the micro-element \( dl \), one can easily obtain the pressure putting on the spool surface given by

\[
P_0 = \frac{F_0}{\Delta s} = \frac{F_0}{Ddl} = \frac{T}{D \cdot R}. \tag{2}
\]

where \( D \) is the fiber cable diameter and \( \Delta s = D dl \) is the micro-element area.

![Fig. 1. Schematic of the fiber cable winding device, where weight \( G \) provides a constant tension force \( T \) to the fiber.](image)

![Fig. 2. Force analysis of the fiber cable spool on which just one layer of fiber cable was wound.](image)

Based on the above analysis, the tension force exerted on every fiber cable is same as \( T \) when one neglects the spool strain and the friction force between fiber cable layers. In this case, through a simple analysis and deduction, one can express the pressure exerted on the \( m \)-th fiber cable layer, \( P_m \), for the total layer on the spool, \( n \), in the following form:

\[
P_m = \frac{1}{D} \sum_{i=m}^{n-1} \frac{T}{R + iD}, \tag{3}
\]

where \( D = 3^{1/2} \times D/2 \) is the distance between the middle of two layers.

2.2. Considering radial direction shrink of fiber cable spool

In practical situation, the pressure from outer fiber cable layers can cause the inner cables align more tightly and deformation. So, the distance between cable layers will decrease. Then, a shrink of the cable spool along the radial direction to its center will happen. Obviously, such a shrink may inevitably lead to a change in the axial direction tension force and the radial direction pressure of the inner cable. Let us take the \( m \)-th cable layer as an example, the radial shrink strain can be expressed as

\[
e_{mr} = \frac{\Delta R}{R + mD} = \frac{P_m}{\pi \cdot r}. \tag{4}
\]

where \( r \) is the radial direction shrinking coefficient of the spool, \( \Delta R \) is the radial direction shrink of the \( m \)-th cable layer. Note that this radial direction shrinking may result in the changes in the axial length, the axial tension force and the radial direction pressure of the fiber cable. Let us explain these three changes one by one. Firstly, the axial shrink of the fiber cable in the \( m \)-th layer, \( \Delta l_m \), denoted by the fiber length change within one winding circle, can be calculated as

\[
\Delta l_m = 2\pi \Delta R = 2\pi \frac{R + mD}{\pi \cdot r} P_m. \tag{5}
\]

Secondly, the axial tension force of the fiber cable in the \( m \)-th layer, denoted by \( T_m \), is accordingly changed from the constant \( T \) to \( T + \Delta T_m \). It should be pointed out that, for the radial shrink case, \( \Delta T_m \) is negative in our simulations. And it can be obtained through the relation [15]

\[
\frac{\Delta T_m}{S} = E \frac{\Delta l_m}{l_m}, \tag{6}
\]

where \( E \) and \( S \) are the axial elasticity modulus and the cross cutting area of the fiber cable, respectively, and \( l_m \) is the one-winding-circle perimeter of the fiber cable in the \( m \)-th layer as given by

\[
l_m = 2\pi (R + mD). \tag{7}
\]

Inserting Eqs. (5) and (7) into Eq. (6), one can obtain the axial actual tension force of the fiber cable in the \( m \)-th layer in the following form:

\[
T_m = \begin{cases} T + \Delta T_m = T + \frac{ES}{\pi r} P_m, & (m = 1, 2, \ldots, n-1), \\ T, & (m = n) \end{cases} \tag{8}
\]

Taking into account that different layer has different tension force due to the radial shrink of the spool (as given by Eq. (8)), one can re-write Eq. (3) as

\[
P_m = \frac{1}{D} \sum_{i=m}^{n-1} \frac{T_{i+1}}{R + iD}. \tag{9}
\]

Using Eqs. (8) and (9) and gradually iterating from the outer layer (the \( n \)-th layer, \( P_n = 0 \)) to the inner layer, one can achieve, after a series of interminable but not difficult deductions, the
radial pressure of the m-th fiber cable layer as

\[
P_m = \frac{T}{D} \sum_{i=1}^{n-1} \frac{1}{R + iD} - \left( \frac{ES}{\pi r} \right) \left( \frac{1}{D} \right)^2 T \sum_{i=1}^{n-2} \frac{1}{R + jD} \left( \sum_{j=i+1}^{n-1} \frac{1}{R + jD} \right) + \left( \frac{ES}{\pi r} \right) \left( \frac{1}{D} \right)^3 T \sum_{i=1}^{n-3} \frac{1}{R + iD} \left( \sum_{j=i+1}^{n-2} \frac{1}{R + jD} \right) + \cdots + \left( \frac{ES}{\pi r} \right) \left( \frac{1}{D} \right)^{n-2} T \sum_{i=1}^{2} \frac{1}{R + iD} \left( \sum_{j=i+1}^{n-1} \frac{1}{R + jD} \right)
\]

\[
- \left( \frac{ES}{\pi r} \right) \left( \frac{1}{D} \right)^{n-3} T \sum_{i=1}^{n-4} \frac{1}{R + iD} \left( \sum_{j=i+1}^{n-3} \frac{1}{R + jD} \right) \times \left( \sum_{z=k+1}^{n-1} \frac{1}{R + 2D} \right) + \cdots + \left( \frac{ES}{\pi r} \right) \left( \frac{1}{D} \right)^{n-1} T \sum_{i=1}^{n-1} \frac{1}{R + iD} \left( \sum_{j=i+1}^{n-2} \frac{1}{R + jD} \right)
\]

We emphasize that Eq. (10) has already included the above mentioned third change, i.e., the radial direction pressure of the fiber cable, results from the radial direction shrink effect. And, in what follows, we will indicate that Eq. (10) can be further simplified when one considers the related parameters according to actual fiber cable spool system. Based on the actual situation, the parameters in Eq. (10) are listed here: \( T = 200 \text{N}, D = 340 \mu m, R = 0.05 \text{mm}, \lambda_0 = 1.30 \times 10^{11} \text{ Pa}, E = 7.452 \times 10^{10} \text{ Pa}. \) Based on the numerical relationship of \( R > D \), it is easy to find that from the third term in Eq. (10) each term can be regarded as an infinitesimal compared to its previous adjacency term. Therefore, we may approximate Eq. (10) by just keeping its first two terms as given by

\[
P_m = \frac{T}{D} \sum_{i=1}^{n-1} \frac{1}{R + iD} - \left( \frac{ES}{\pi r} \right) \left( \frac{1}{D} \right)^2 T \sum_{i=1}^{n-2} \frac{1}{R + jD} \left( \sum_{j=i+1}^{n-1} \frac{1}{R + jD} \right).
\]

Eq. (11) means that the radial pressure of fiber cable layer is quite insensitive to the radial shrink of the fiber cable spool. However, the axial shrink of the fiber cable originated from the radial shrink of the fiber cable spool will result in a sensitive change in the FBG’s central wavelength. Note that we will discuss this point in Section 2.3 in detail.

### 2.3. Considering friction force between fiber cable layers

Considering the friction force between fiber cable layers, the axial direction tension force of the fiber cable in the m-th layer can be given by

\[
T = T - \mu P_m \pi (R + mD)D,
\]

where \( \mu \) is the static friction coefficient. Replacing \( T \) in Eq. (11) with \( P \) expressed by Eq. (12), we thus deduce the radial pressure of the m-th fiber cable layer as given by

\[
P_m = \frac{\sum_{i=1}^{n-1} \frac{1}{R + iD} - \left( \frac{ES}{\pi r} \right) \left( \frac{1}{D} \right)^2 \sum_{i=1}^{n-2} \frac{1}{R + jD} \left( \sum_{j=i+1}^{n-1} \frac{1}{R + jD} \right)}{1 + \mu \pi (R + mD) \sum_{i=1}^{n-1} \frac{1}{R + iD} - \left( \frac{ES}{\pi r} \right) \left( \frac{1}{D} \right)^2 \mu \pi (R + mD) \sum_{i=1}^{n-2} \frac{1}{R + jD} \left( \sum_{j=i+1}^{n-1} \frac{1}{R + jD} \right)}
\]

In the experiment, in order to measure the radial pressure distribution with the fiber cable layer, a serial of FBGs with different Bragg wavelengths were fused into the fiber cable as pressure sensors. It is well-known that FBG’s Bragg (center) wavelength change resulted from the axial direction strain is given through the following relation [15]:

\[
\Delta \lambda_{BR} = \frac{1}{\lambda_0} \left( 1 - \frac{P_m}{\pi r} \right) \Delta a_2 \text{m}.
\]

where \( \lambda_0 \) is the FBG’s Bragg wavelength at free state, \( P_m \) is the fiber effective elastico-optic coefficient, \( \Delta a_2 \text{m} = \Delta a_2 / \lambda_0 \) is the FBG’s axial direction strain. Note that \( \Delta a_2 \) and \( \lambda_m \) have already defined by Eqs. (5) and (7), respectively. The subscript ‘z’ indicates the axial direction of the fiber cable. Inserting Eqs. (5) and (7) into Eq. (14) and considering the sign of \( \lambda_m \), the change in Bragg wavelength of FBG in the m-th fiber layer is then given by

\[
\Delta \lambda_{BR} = -\frac{1}{\lambda_0} \left( 1 - \frac{P_m}{\pi r} \right) \Delta a_2 \text{m}.
\]

On the other hand, considering that the radial direction pressure \( P_m \) is exerting on the FBG, this may lead to an additional shift to the FBG Bragg wavelength, as given by [16]

\[
\Delta \lambda_{BR} = -k_r \Delta p_m,
\]

where \( k_r \) is the radial pressure sensitivity coefficient of FBG. Note here we use the subscript ‘r’ to indicate the radial direction of the fiber cable. Obviously, under both the axial-direction and the radial-direction strains, the change in Bragg wavelength of FBG can be calculated as

\[
\Delta \lambda = \Delta \lambda_{BR} \Delta \lambda_{BR} = -\frac{1}{\lambda_0} \left( 1 - \frac{P_m}{\pi r} \right) + k_r \lambda_0 \Delta p_m.
\]

where \( \lambda_0 \) is the radial direction shrinking coefficient. Although \( \lambda_0 \) is usually a constant, for our case it varies as the fiber winds on the spool since the spatial gap between fiber cable layers always changes during the winding process. Considering this effect, Eq. (17) should be revised if one takes into account the actual situation of fiber winding process. The dependence can be explained as follows. On one hand, with the increase of \( m \), the radial direction shrinkage of the m-th fiber cable layer becomes greater and greater. On the other hand, with the increase of the total layer number \( n \), the inner fiber cable layer becomes increasingly tight and the axial direction shrink becomes increasingly smaller. Taking into account this two factors, the shrinkage coefficient, \( \lambda_0 \), needs a modification to satisfy the practical situation. Based on the above analysis, we hereby suppose that the radial shrink coefficient depends on \( m \) and \( n \) through \( \lambda_0 / m \times n \). In this case, Eq. (17) should be modified and denoted as

\[
\Delta \lambda = \left( \frac{1}{\lambda_0 / m} \times n \right) k_r \lambda_0 \Delta p_m.
\]

Eq (18) denotes the Bragg wavelength shift of the FBG in the m-th fiber cable layer caused by both the axial- and the radial-direction strains, where the total layers wound on the fiber spool is \( n \). The comparison between the experimental results and the simulations, showing in the following Sections 3 and 4, indicate that such a modification to the radial shrink coefficient is basically correct.

### 3. Experimental results

In order to measure the static pressure at different layers of the fiber cable spool, we need to embed the sensing elements (FBGs) into the fiber cable. Fig. 3 depicts the schematic diagram to show how the FBGs are arranged along the fiber cable. This corresponds to a fiber cable we fabricated and used in the experiment, where 11 FBGs were fused into the fiber cable with a length of 2 km. The Bragg wavelengths of the 11 FBGs are from 1534 nm to 1554 nm with a wavelength interval of 2 nm.

![Diagram of the FBGs fused into the fiber cable](image-url)
The fabrication process of the FBG-embedded fiber cable can be simply described as follows: firstly, the first FBG with the wavelength of 1534 nm was fused at 500 m location, as shown in Fig. 3, secondly, the other 10 FBGs with corresponding wavelengths were fused into the fiber with a distance interval of 50 m and thirdly, the last 1000 m fiber was fused. Finally, such a FBG-embedded fiber was re-coated and recovered to a fiber cable through a special treatment process.

Fig. 4 shows the schematic and relative positions of the eleven FBGs in the spool. Note that for the first layer the wound cable length is totally 78 m where the cable is wrapped on the spool with a radius of 0.05 m. In order to ensure that every fiber cable layer to be tightly wound on the spool thus to prevent the layer collapse, during the winding process, the two ends of each layer are gradually shrunk to the center. The black small square marks, with the corresponding number labels, indicate the positions of the 11 FBGs in the spool. For example, the first FBG is located on the 7th layer when the wound cable length reaches 500 m.

We tested the whole winding process. When it reaches the 7th fiber cable layer \((n = 7)\), the first FBG was wound on the spool. At this moment, the Bragg wavelengths of the FBGs string were demodulated and the first FBG’s Bragg wavelength was obtained. And then, with a continuous winding, the second FBG was wound on the spool when the cable layer number was 8 \((n = 8)\). At this moment, the Bragg wavelengths of the FBG string were demodulated again and the first and the second FBG’s Bragg wavelengths were obtained.

Fig. 5(a) shows the Bragg wavelengths variations of the eleven FBGs during the whole experimental winding process. We can see that the FBG’s Bragg wavelengths in the fiber cable layers vary very quickly at the beginning and then become gently. This feature is consistent to the fact that the inner cable layers become gradually tight and stable as the cable layers increase. Note that, as compared with other FBGs, the 4th FBG’s Bragg wavelength variation is distinct. This is because that the 4th FBG is located near the edge of the layer, it is influenced by fewer layers above it. (see Fig. 4).

4. Theoretical simulation

It is seen from Eq. (18) that the Bragg wavelength shift of FBG depends on two factors. Firstly, the radial direction shrink of the fiber cable spool causes every layer FBG axial direction strain. Secondly, the pressure from the outer fiber cable layers causes the radial direction strain of the FBG located in the inner layer. Using Eqs. (13) and (18) we simulated the Bragg wavelength shifts of FBGs and the pressures for fiber cable layers from the 7th layer to the 15th one, supposing that there are 50 layers being wound on the spool. The Bragg wavelengths of FBGs are from 1534 nm to 1554 nm with an interval of 2 nm. The parameters used in simulations are the actual experimental parameters. i.e., \(P_r=0.22\), \(\alpha_r=1.30 \times 10^{11}\) Pa, \(k_{pr}=8.38 \times 10^{-13}\) Pa\(^{-1}\), \(R=0.05\) m, \(D=340\) \(\mu\)m, \(T=200\) N, \(\mu=0.00625\).
Secondly, the modification of the shrinkage coefficient, inevitably has a fluctuation during the process of fiber winding. The radial direction shrink of the spool with the increase of the fiber cable layer number might further increases. Here we select \( m \) from 7 to 15 and \( n \) from 0 to 50 as example. It seems that \( P_m \) varies very quickly at the beginning and then becomes gently. The fiber cable layer pressure \( P_m \) on the fiber cable layer number \( n \) is shown in Fig. 5(b). It is seen that the theoretical variation of the FBG Bragg wavelength with the fiber cable layer number \( n \). Here we select \( m \) from 7 to 15 and \( n \) from 0 to 50 as example. It seems that \( P_m \) will go to convergence as \( n \) further increases.

In fact, the winding process is a dynamic stabilizing process, i.e., the inner cables become gradually tight and stable owing to the radial direction shrink of the spool with the increase of the layers. Therefore the shifts of Bragg wavelengths and the variations of the pressures become stable gradually.

Based on the experimental data of Bragg wavelengths variation of the eleven FBGs shown in Fig. 5(a), using Eq. (18) we achieve the relationship between radial direction pressure \( P_m \) of the fiber cable layer and the fiber cable layer number \( n \), as shown in Fig. 7. It is seen that Fig. 7 basically coincides with the simulation result (see Fig. 6).

5. Conclusion

In conclusion, we established a theoretical model for describing the inner static pressure among fiber cable layers in the spool of the FOG-M based on the force analysis of the system. Both the simulations and the experiments were conducted. Simulations indicate that the radial pressure of fiber cable layer in the spool asymptotically converges as the number of fiber layers increase. At the same time, the FBG’s Bragg wavelength in the fiber cable layer varies very quickly at the beginning and then becomes gently. The Bragg wavelength variation of FBG is resulted from a combined effect including both the FBG axial strain caused by the spool radial shrink and the FBG radial strain caused by the pressure from outer cable layers during the winding process. The simulation results agree qualitatively with the experimental ones. This technology provides us a real-time method to monitor the pressure of the fiber cable layer during the cable winding process. This research has a significant reference in many areas such as the fiber cable spool winding, the pressure measurement of the fiber cable spool, the fiber cable spool design.

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