Intrinsic low-frequency variability and predictability of the Kuroshio Current and of its extension

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Intrinsic low-frequency variability and predictability of the Kuroshio Current and of its extension

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Investigations of the intrinsic low-frequency variability and predictability of the Kuroshio Current and of its extension jet (the Kuroshio Extension, KE) are reviewed. The Kuroshio and KE in the North Pacific constitute a western boundary current system of great relevance from climatological and ecological viewpoints. Both the Kuroshio south of Japan and the KE display remarkable changes of bimodal character on interannual time scales that are believed to be intrinsic, i.e., basically generated by nonlinear oceanic mechanisms rather than by direct atmospheric forcing. Model studies of the Kuroshio and KE with climatological forcing are thus reviewed. Moreover, as these changes are chaotic, their predictability requires peculiar mathematical approaches: theoretical results concerning this important issue are therefore reviewed as well. Model studies aimed at determining the optimal precursors and optimally growing initial errors for the Kuroshio are described. Techniques based on Lyapunov exponents (including their Lagrangian extension) and on data assimilation techniques (namely, sequential importance sampling using a particle-filtering approach) are reviewed for the KE. The key problem of how to identify the areas where targeted observations can improve the forecast is also addressed. The role of wind forcing in triggering the KE oscillations is finally considered.

Keywords: physical oceanography; ocean modelling; western boundary currents; Kuroshio; low-frequency variability

1. Introduction

Western boundary currents (WBCs) are the most intense currents of the world’s oceans. They are wind-driven in the oceanic interior and flow along the western boundaries of the oceans, where they are intensified because of the sphericity of the earth, which generates the so-called planetary beta effect [1–4]. The Gulf Stream and Kuroshio are notable examples of WBCs belonging to the subtropical gyres of the North Atlantic and Pacific Oceans, respectively; other important cases are the Oyashio, Agulhas, East Australian and Brazil Currents. The effect of WBCs and of their respective extensions (i.e., of those currents that flow as free inertial meandering jets after separation from the coast) on climate is well known to be due to their huge meridional mass and heat transport and corresponding vigorous air-sea interactions (e.g., [5,6]), and to the role some of them play in sustaining the global meridional overturning circulation (e.g., [7,8]). International projects such as the US Climate Variability and Predictability Research Program (CLIVAR; http://www.usclivar.org/wbc.php) and the Kuroshio Extension System Study (KESS;
http://uskess.org/) have been devoted to investigating the main WBC systems both experimentally and theoretically.

WBCs are also very important from an ecological viewpoint. They enhance primary productivity through their transport of nutrients back to the euphotic layer from which they continuously leak to deeper waters [9], and are major spawning grounds for fish species [10,11]. Several commercially valuable types of organisms, like squid, eels and salmon can be rapidly transported over long distances within WBCs with minimum energy expenditure between their reproduction and feeding areas [12]. In such cases, the strong WBCs allow a more efficient use of resources and can also play a relevant role in gene flow between opposite sides of ocean basins [9].

Among the various WBCs, the Kuroshio Current (Kuroshio in brief) and the corresponding Kuroshio-Oyashio Extension (KE heretofore) constitute a major WBC system (e.g., Qiu [13]). The Kuroshio flows south of the islands of Japan and, after separating from the coast at about 35°C, it flows almost directly eastwards into the open ocean giving rise, together with the Oyashio, to the KE (Figure 1 adapted from [14]), a strong inertial jet characterized by large meanders and energetic pinched-off eddies. The Kuroshio system as a whole has been observed to have multiple paths [15]. The path taken by the Kuroshio south of Japan is divided into two categories, the Large Meander (LM, or meandering path), where the current meanders offshore and then returns to the coast before its separation point, and the Non-Large Meander (NLM, or alongshore path), where the current stays close to the coast until its separation point [16,17]. The duration of the LM and NLM Kuroshio paths is not well defined but varies between a few years to a decade (it is customary to define such oceanic variability as low-frequency, so this definition is adopted here). Also, the KE displays a low-frequency variability on the decadal time scale that connects a zonally elongated, fairly stable and energetic meandering jet and a much weaker, highly variable and convoluted jet with a reduced zonal penetration

![Figure 1](image_url). Mean sea surface dynamic topography relative to 1000 dbar (in cm) based on climatological data sets. Line P10 denotes the lowered acoustic Doppler current profiler (ADCP) section of the world ocean circulation experiment (WOCE) P10 cruise (from Qiu and Miao [14]).
(the so-called elongated and contracted modes, respectively [18]). The low-frequency variability of both the Kuroshio and KE are therefore often referred to as being bimodal.

A fundamental issue in this framework concerns the understanding of the physical mechanisms that cause the Kuroshio and KE low-frequency variability. This is a very complex problem that involves nonlinear ocean-atmosphere interactions and feedbacks, barotropic and baroclinic instability, nonlinear eddy-mean flow interactions, Rossby wave propagation, interaction with topographic and coastal features, etc. There is a part of the low-frequency oceanic variability denoted as ‘intrinsic’, for which physical mechanisms internal to the ocean system play a more important role than that of the atmospheric forcing. In the last two decades sound theoretical and modelling arguments have been provided supporting the hypothesis that intrinsic mechanisms play a central role in the low-frequency variability of the wind-driven ocean circulation, with specific reference to WBC systems [19,20]. The Kuroshio and KE, in particular, were found to be ideal examples of this behaviour (e.g., [14,21–23]). On the other hand, such variability is basically due to complex nonlinear instability processes leading to chaotic behaviour, which makes its predictability a very peculiar and delicate problem. This was attacked recently using different mathematical approaches (e.g., [23–26]).

In this article we review the main observational and modelling studies on the low-frequency variability of the Kuroshio and KE, with an emphasis on its intrinsic component; theoretical and modelling studies on the predictability of the variability are also reviewed. The paper is organized as follows. In Sections 2 and 3 the low-frequency variability of the Kuroshio and KE is considered, respectively. In Sections 2.1 and 3.1 the respective observational evidence is presented, while in Sections 2.2 and 3.2–3.5 theoretical interpretations based on model results are discussed. In Sections 4 and 5 the predictability of the Kuroshio and KE low-frequency variability is considered, respectively. For the Kuroshio, applications of the conditional nonlinear optimal perturbation method [27] to evaluate the optimal precursors, and the optimally growing initial errors [27–30] are discussed in Sections 4.1–4.2. For the KE, applications of the Lyapunov exponents [23] (also in a Lagrangian form) are discussed in Section 5.1, while data assimilation methods [26] based on the particle-filtering technique [31] are presented in Section 5.2. In Section 5.3 recent modelling results [32] concerning the role of wind forcing in the synchronization of the KE bimodality with the main atmospheric modes of variability in the North Pacific are summarized, and related issues are discussed. Finally, in Section 6 conclusions are drawn.

2. Low-frequency variability of the Kuroshio

2.1 Observations

The Kuroshio Current is the northward branch of the North Pacific subtropical gyre and is one of the major WBCs of the world’s oceans. It is a narrow (100 km) and strong \((O(1 \text{ m s}^{-1}))\) current extending vertically above 1000 m (Figure 2a), flowing roughly from 15°N to 35°N, where it separates from the coast (Figure 1). The volume transport across the WOCE P10 line, just before separation (Figures 1, 2a), is 130 Sv [33], which is more than twice as large as the maximum Sverdrup transport in the subtropical North Pacific [34]. The excess transport is due to the presence of the westward recirculating flow south of the Kuroshio, also known as the Kuroshio Countercurrent, with a strong barotropic component reaching a total transport of 80 Sv.

Important low-frequency variations of the shape and position of the Kuroshio Current south of Japan are well known to occur and have been studied for many decades, as they have a great influence on local fisheries, ship navigation, marine resources, etc. The
Figure 2. (a) Eastward velocity profile from lowered ADCP measurements on the WOCE P10 line of Figure 1 (from Qiu and Miao [14]). (b) Typical paths of the Kuroshio: nNLM is the nearshore non-large meander (NLM) path, oNLM is the offshore NLM path; tLM is the typical large-meander path (from Kawabe [17]).
existence of two major Kuroshio paths, namely the LM and NLM states, was already recognized in the 1960s (e.g., [35]) and justifies the expression ‘Kuroshio bimodality’. Figure 2(b) shows a schematic representation of the Kuroshio paths [17]: the NLM state can in turn be divided into two different sub-states, namely the nearshore and offshore NLM paths. Kawabe [36] analysed the transition processes connecting the three states using sea level and temperature data. The NLM to LM transition occurs within few months due to a generation of a small meander of the Kuroshio southeast of Kyushu. The LM to NLM transition accompanies an eastward shift of the meander and its east-west oscillation over the Izu Ridge during a decay of the meander. During this transition the evolution is relatively gradual, and was already recognized in the early papers (e.g., [36]) to be hardly predictable.

Figures 3(a,b) from Taft [15] provide a classical picture of the different Kuroshio bimodal path patterns. Figures 3(c,d) from Wang et al. [37] show examples of the two paths based on the more recent satellite altimeter data: these high resolution (1/3°×1/3°) sea surface height (SSH) data were produced and distributed by Archiving, Validation and Interpretation of Satellite Oceanographic data (AVISO). The Kuroshio was in a NLM state in July 2003–June 2004 (Figure 3c), while it was in a LM state in July 2004–June 2005 (Figure 3d). These observations confirm that each path can persist from a few years to a decade. In contrast, the actual transition between the two typical paths can take place within several months.

Figure 3. Examples of the bimodal Kuroshio paths south of Japan: (a) straight paths, (b) meander paths (adapted from Taft [15] and Qiu and Miao [14]). Mean SSH altimetric fields from AVISO for the period of (c) July 2003 to June 2004 and (d) July 2004 to June 2005 (contour intervals are 10 cm, units in cm, from Wang et al. [37]).
2.2 Models and theoretical explanation

Understanding the Kuroshio bimodality is a very complex dynamical problem. First of all it should be recognized that this is a highly nonlinear phenomenon in which instability processes must play an important role: simplistic explanations based on linear, local wind effects are therefore to be ruled out. As Qiu and Miao [14] pointed out, for the Kuroshio to smoothly rejoin the Sverdrup interior flow at the higher latitude of \(30^\circ - 35^\circ N\), the anomalously low potential vorticity (PV) acquired by the Kuroshio in the south has to be removed by either dissipative and/or nonlinear forces along its western boundary path; however, dissipative forces alone are not sufficient to remove the low PV anomalies [38]. The consequence of the Kuroshio’s inability to effectively diffuse the PV anomalies along its path is the accumulation of the low PV water in the northwestern corner of the subtropical gyre, which generates a mean anticyclonic recirculation gyre (Section 2.1), and provides an energy source for flow instability.

In any instability process and its nonlinear development the intensity of the flow plays a fundamental role. Thus, as for any WBC, low-frequency variations of the Kuroshio transport are expected to depend on changes in the total meridional Sverdrup transport in the oceanic interior, which is in turn determined by large-scale low-frequency variations of the wind stress curl. This dependence can, however, be quite complex (e.g., [39]). Moreover, no simple relation can be established between the Kuroshio strength and its path: for example, Qiu and Miao [14] noted that the connection between the Kuroshio path and the magnitude and/or the temporal change of the upstream inflow transport is not conclusive based on the long-term available observational data. Therefore they argued that the observed oscillations between the straight and meander paths are not necessarily controlled deterministically by the temporal changes in the upstream Kuroshio transport. Hence, nonlinear intrinsic oceanic mechanisms are expected to generate the observed Kuroshio low-frequency bimodal variability, whose investigation requires the use of nonlinear ocean models.

To this respect, Qiu and Miao [14] adopted a two-layer primitive equation ocean circulation model that incorporated the realistic North Pacific coastlines and bottom topography and was driven by observed monthly climatological wind stresses without any interannual variability, so that all changes in the ocean response had to be due to nonlinear intrinsic oceanic mechanisms. Their model did in fact produce chaotic alternations of the Kuroshio’s two states, which were proposed to be due to a self-sustained internal mechanism involving the evolution of the southern recirculation gyre and the instability of the Kuroshio system. Figure 4(a) shows the modelled Kuroshio path index defined as the mean distance of the Kuroshio axis from the Japan coast (132°–140°E), while Figures 4(b,c) show composite modelled upper-layer thickness fields for the NLM and LM paths, respectively.

Other model studies yield a Kuroshio bimodal behaviour under forcing without interannual variations [22,37,40,41]. Pierini [22] forced his shallow-water reduced-gravity model with steady winds and focused mainly on the KE bimodality (see Sections 3.4 and 5.1 for a detailed discussion), but NLM and LM states of the Kuroshio south of Japan similar to those observed did emerge. Douglass et al. [40] forced the Parallel Ocean Program (POP) model with an annually repeating atmosphere, and the obtained bimodality was found to be comparable in timing and spatial extent to observed features in the region. Wang et al. [37] were able to simulate correctly the Kuroshio path variations with a reduced-gravity shallow-water model under monthly averaged climatological wind forcing (see Section 4.2 for a detailed discussion). By means of a two-way nested ocean
general circulation model (OGCM) driven by a repeat annual cycle forcing, Kurogi et al. [41] obtained a realistic Kuroshio bimodality, which they analysed using vorticity diagnostics.

In conclusion, all these climatologically forced model studies produce chaotic alternations between LM and NLM Kuroshio paths in agreement with observations, and thus strongly support the hypothesis that the Kuroshio bimodality is an intrinsic oceanic phenomenon. Note that the model implementations [22,37] were based on reduced-gravity equations in which, therefore, baroclinic instability was absent: thus, barotropic instability (present in all models) is a good candidate for the basic instability mechanism sustaining the Kuroshio bimodality.

3. Low-frequency variability of the Kuroshio Extension

3.1 Observations

As we have already seen, the KE is the highly stratified eastward-flowing free inertial meandering jet formed when the confluent Kuroshio and Oyashio WBCs detach from the Japanese coastline at about 35°N (Figure 1). The KE constitutes, therefore, a front separating the warm subtropical and cold subpolar waters of the North Pacific Ocean (e.g., Qiu et al. [42]). Figure 5 shows the frontal, virtually reduced-gravity structure of the jet, with currents confined in the upper 500 m reaching locally the velocity of 2 m s⁻¹, an extreme oceanic value. The KE is an ocean region where one of the most intense air-sea heat exchanges takes place. It is also the region of the North Pacific with the highest eddy
kinetic energy level [43], and in which large-scale interannual changes lead to large temperature anomalies that are capable of enhancing the variability of the midlatitude coupled ocean-atmosphere system, thus, strongly affecting North American climate [43,44]. It is therefore very relevant, from a climatic viewpoint, to investigate the variability of the KE system. In addition, monitoring and possibly predicting the KE path is very important also for local fisheries, and hence local economies, since the position of the jet strongly determines the regions where phytoplankton, and hence fish, is located. The KE and its variability have been the subject of a number of experimental studies, like the observational programme of the Intergovernmental Oceanographic Commission (IOC) of the United Nations Educational, Scientific and Cultural Organization (UNESCO) sub-commission for the western Pacific (WESTPAC) at 152°E [45], the Kuroshio Extension Region Experiment (KERE) at 143°E [46], and the KESS at 146°E [47].

The KE low-frequency variability is one of the most fascinating phenomena occurring in the world’s oceans. Based on the first eight years of TOPEX/Poseidon altimeter measurements, Qiu [18] described the transition from a KE state called ‘elongated’, characterized by a zonally extended jet with two relatively stable large meanders and an intensified southern recirculation gyre, to a ‘contracted’ state, characterized by a weaker, more convoluted jet with a reduced zonal penetration. In two subsequent papers, Qiu and Chen [48,49] extended the previous analysis by using updated altimeter data sets. Figure 6 shows the time series of the upstream KE path length $L_{KE}$ documenting two full KE

Figure 6. Upper panel: upstream KE path length derived from altimeter data (updated time series to Qiu and Chen [48,49], courtesy of B. Qiu). Lower panels: maps of yearly averaged SSH altimetric fields from AVISO (contour intervals are 10 cm with the thick lines denoting the 170-cm contours: adapted from Qiu and Chen [49]).
cycles, i.e., two transitions like the one just described, followed by two opposite transitions that connect in a very different fashion the contracted state to the elongated state; this is why this behaviour is usually denoted as bimodal, although more than two states could be identified. Using the measure $L_{KE}$, the elongated state is recognizable by a signal that is smaller than the mean and with small variance, whereas for the contracted state this parameter is much more variable and has a larger average value. The panels of Figure 6 are yearly averaged SSH altimetric maps showing the subsurface geostrophic currents at different times. Next in Sections 3.2–3.5 different theories aimed at explaining the observed KE bimodality will be reviewed, with a focus on the crucial role of intrinsic oceanic mechanisms.

3.2 External variability: response to wind variations

In the so-called Rossby wave view [50], wind stress changes over the North Pacific are thought to be crucial for the transitions between the contracted and elongated KE states. The wind stress over the North Pacific varies on a decadal time scale, for example, associated with the Pacific Decadal Oscillation (PDO) which has its centre of action around 160°W [51]. When the PDO index is positive (negative), the Aleutian Low shifts southwards (northwards) and negative (positive) SSH anomalies are generated through Ekman divergence. Once generated, they propagate westward with the speed of baroclinic Rossby waves $c_R$, which, at typical KE latitudes (~30°N), is about 4 cm s$^{-1}$.

The linear vorticity equation under the long-wave approximation of the SSH anomaly field $h'(x,y,t)$ in local Cartesian coordinates $(x,y)$ is given by

$$\frac{\partial h'}{\partial t} - c_R \frac{\partial h'}{\partial x} = - \frac{g'}{\rho_0 g f} \left( \frac{\partial \tau^x}{\partial y} - \frac{\partial \tau^y}{\partial x} \right)$$

(1)

where $g$ is the gravitational acceleration, $g' = \frac{\Delta g}{g}$ the reduced gravity, $\rho_0$ a reference density, $f$ the Coriolis parameter, and $(\tau^x, \tau^y)$ the wind-stress field. When the wind stress is given, the SSH anomaly field can be determined along Rossby wave characteristics. This response was calculated by Qiu and Chen [48] and is shown here in Figure 7(a), together with the observed SSH anomaly field from altimeter data (Figure 7b). Indeed, variations in the large-scale wind-stress forcing lead to Rossby waves and large-scale SSH anomalies with similarities between model results and observations. The amplitudes of the observed SSH anomalies, however, are larger than those from the Rossby wave model (Equation 1) and show variability on smaller scales.

When the SSH anomalies from the Rossby-wave model (Equation 1) are superimposed on the mean dynamic topography [52], the biennially averaged SSH fields as in Figure 8(a) are obtained. Comparison with the corresponding fields derived from altimeter data (Figure 8b) shows that the intensity of the modelled recirculation gyre south of the main axis of the jet (thick black line) appears in phase with the alternation of the elongated and contracted states of the KE. However, the modelled gyre is always much weaker than the observed one and, what is most striking, the jet axis and the nearby isolines are virtually time independent in the model results, while they undergo substantial changes during the observed bimodal cycle. It is clear from these results that additional processes are needed to explain the bimodality of the KE (in Section 5.3 we will reinterpret the role of Rossby waves in light of new model results).
The influence of the mesoscale eddy field therefore is considered to be important [49]. In taking a more southward path, the KE interacts with the underlying topography, in particular the Shatsky Rise, to lead to an intensified mesoscale eddy field. Qiu and Chen [49] show that the changes in this eddy field lead to an intensified southern recirculation gyre and hence push the KE northward, a motion which is exacerbated by the arrival of positive SSH anomalies which have been generated during the negative phase of the PDO [53].

In summary, in this interpretation of the observations the low-frequency variability of the KE is mainly forced by wind anomalies over the central and eastern North Pacific. Rossby waves generated by time-varying winds produce a spatially broad variability in the KE region that is in synchrony with the much stronger, spatially sharper bimodal variability observed in altimeter data. Apart from any consideration concerning possible interactions between the two kinds of low-frequency variability (see Section 5), it is clear, however, that wind-driven Rossby waves can by no means account for the vigorous bimodal variability so clearly evidenced in observations. But, as we will see in Sections 3.5 and 5.3, they do play a fundamental role in the timing of the KE bimodal cycle.

3.3 Intrinsic variability: quasi-geostrophic conceptual models

In the intrinsic variability view, the KE bimodal variability is generated through dynamical mechanisms internal to the ocean system. In order to unequivocally recognize the
Figure 8. Biennially averaged SSH fields with the SSH anomalies (a) hindcast by the linear vorticity model and (b) observed by the satellite altimeters. The mean SSH field in (a) is based on Teague et al. [52] and thick lines denote the 170-cm contours (from Qiu and Chen [48]).
intrinsic origin of the low-frequency variability in a particular model, a stationary wind forcing is used. In this case, all changes found in the equilibrium (asymptotic) response are necessarily due to mechanisms internal to the model ocean.

The starting point is the theory of the homogeneous steady wind-driven ocean circulation [1,2,54] which is one of the cornerstones in physical oceanography. One of the simplest situations within this theory is that of an active layer of ocean water with constant density $\rho$ located in a rectangular ocean basin. Below this layer, with equilibrium thickness $H$, there is a very deep motionless layer of density $\rho + \Delta \rho$. The flows are considered on a midlatitude $\beta$-plane with Coriolis parameter $f = f_0 + \beta y$.

Let the flow be characterized by a horizontal length scale $L$ and a horizontal velocity scale $U$. When the Rossby number $e = U/(f_0 L)$ is small, quasi-geostrophic theory is an adequate description of the large-scale flow [3]. Let $c$ indicate the geostrophic streamfunction in the horizontal plane: then the zonal velocity $u$, the meridional velocity $v$ and the vertical component of the relative vorticity $\zeta$ are given by $u = -\partial \psi / \partial y$, $v = \partial \psi / \partial x$ and $\zeta = \partial v / \partial x - \partial u / \partial y = \nabla^2 \psi$, respectively. When the flow is driven by a wind stress $\tau$, the governing equation in this theory is the (equivalent) barotropic vorticity equation, given by

$$\frac{\partial q}{\partial t} + J(\psi, q) = A_H \nabla^4 \psi + \frac{(\nabla \times \tau)_z}{\rho H} \tag{2a}$$

$$q = \nabla^2 \psi - \frac{\psi}{R^2} + \beta y \tag{2b}$$

Here, $q$ is the PV, $g'$ is the reduced gravity, $H$ is the active upper layer thickness, the Jacobian operator $J$ is defined as $J(F, G) = F_x G_y - F_y G_x$ (where the subscripts indicate differentiation), and $R = \sqrt{g' H / f_0}$ is the internal Rossby deformation radius. The quantity $A_H$ represents the turbulent lateral friction coefficient.

In the so-called double-gyre case, flows are considered in a rectangular $L \times B$ basin where the wind-stress forcing is chosen as

$$\tau^x = -\tau_0 \cos(2\pi y/B); \quad \tau^y = 0 \tag{3}$$

with $\tau_0$ a typical amplitude. In this case, the wind stress is symmetric with respect to the mid-axis of the basin. No-slip boundary conditions at the east-west boundaries and free-slip conditions at the north-south boundaries are prescribed, i.e.,

$$x = 0, \ L : \ \psi = 0, \ \frac{\partial \psi}{\partial x} = 0 \tag{4a}$$

$$y = 0, \ B : \ \psi = 0, \ \nabla^2 \psi = 0 \tag{4b}$$

Under a given steady wind-stress forcing, the linear steady quasi-geostrophic theory (neglecting the terms $J(\psi, q)$) predicts a Sverdrup interior flow and a frictional western boundary layer. The linear theory provides a first order explanation of the existence of western boundary currents, such as the Kuroshio. The nonlinear theory is, however, still far from complete. Although the strong effects of inertia on the flows was already shown by [55], the work to determine systematically the solution structure of Equations (2a) and (2b) versus the lateral friction parameter $A_H$ did not start until the mid-1990s [56,57].

For large values of $A_H$, a unique and globally stable flow state for both single- and double-gyre cases is found [58]. To investigate the solution structure of the equations
when $A_H$ is decreased, continuation methods [19] have been used on discretized versions of Equations (2a) and (2b).

The structure of the steady solutions is shown through the bifurcation diagram in Figure 9, where the value of the streamfunction at a point in the southwest part of the domain ($\psi_{SW}$) is plotted versus the Reynolds number $Re = UL/A_H$. At large values of $A_H$ (small $Re$), the anti-symmetric double-gyre flow (at label (a) in Figure 9) is a unique state. When lateral friction is decreased, this flow becomes unstable at the pitchfork bifurcation $P_1$ (near $Re = 30$) and two branches of stable asymmetric states appear for smaller values of $A_H$ (larger $Re$). The solutions on these branches (at locations (b) and (d)) have the jet displaced either southward or northward and are exactly symmetrically related for the same value of $Re$ (streamfunction plots of these solutions can be found in [57]).

The important point from Figure 9 is that multiple stable asymmetric equilibria (here steady flows) can occur under a symmetric wind-stress forcing. The mechanism of this symmetry breaking is due to shear instability and has been explained in detail in [57].

The asymmetric states which arise from $P_1$ also become unstable at larger values of $Re$ due to the occurrence of Hopf bifurcations. The first Hopf bifurcation, $H_1$ in Figure 9, is associated with the destabilization due to a so-called Rossby-basin mode. These modes can be described by a sum of free Rossby waves, where the coefficients are chosen such that the boundary conditions are satisfied. For the gravest Rossby basin mode, the period is about 20 days. At a second Hopf bifurcation, $H_2$ in Figure 9, the asymmetric state destabilizes to a mode which has an interannual period and the perturbations strengthen and weaken the eastward jet during both phases of the oscillation. These interannual, so-called gyre modes do not have their origin in the spectrum of the linear operator related to free Rossby-wave propagation. Simonnet and Dijkstra [59] clarified the spectral origin of the gyre mode, presented a physical mechanism of its propagation and showed the relation between the gyre mode and stationary Rossby waves (see also [60]).

![Figure 9](image-url)
The connection between the first pitchfork bifurcation (indicated by \( P \) in Figure 10), the gyre modes and the occurrence of homoclinic bifurcations was clarified in Simonnet et al. [61] and an overview of the bifurcation behaviour leading to a homoclinic orbit is shown in Figure 10. The symmetry-breaking pitchfork bifurcation \( P \) is responsible for the asymmetric states. The low-frequency gyre modes arise at so-called merging points \( M \) and obtain a positive growth factor at the Hopf bifurcations \( H \). Subsequently, the periodic orbits arising from these Hopf bifurcations on both asymmetric branches connect with the unstable anti-symmetric steady state at the point \( A \); this gives rise to a homoclinic orbit. The type of homoclinic orbit depends on the eigenvalues associated with the linear stability of the symmetric state at the connection point \( A \) [62]. In case there are only real eigenvalues, there is a homoclinic connection of Lorenz type and when the second and third eigenvalues form a complex-conjugate pair, there is a homoclinic bifurcation of Shilnikov type. Simonnet et al. [61] show that both types can occur and that the Shilnikov type is more likely to occur at small \( A_H \), in accordance with the results in [63,64].

The important point from Figure 10 is that low-frequency variability arises spontaneously due to Hopf bifurcations and homoclinic bifurcations, even under a steady wind stress and without mesoscale eddies arising through baroclinic instability. In the regime beyond the homoclinic transitions, variability with a decadal time scale is found to be robust in these models [65–67].

3.4 Intrinsic variability: shallow-water models

The results in the previous section were for quasi-geostrophic models in rectangular basins and one might wonder what happens in reduced-gravity shallow-water models using a more realistic basin geometry and annual-mean observed winds. This issue was investigated in Schmeits and Dijkstra [21] and Pierini [22]. Schmeits and Dijkstra [21] performed numerical bifurcation studies for a part of the North Pacific basin [120°E, 150°W] × [10°N, 55°N] using a horizontal resolution of about 1/2° in a barotropic
shallow-water model. As control parameter, the Ekman number \( E = A_H/(2\Omega r_0^2) \) was used, where \( r_0 \) is the radius of the earth and \( \Omega \) its angular velocity (for model equations and parameter values, see [21]). The bifurcation diagram (Figure 11a), where the maximum northward volume transport (in Sv) is plotted versus \( E \), shows a perturbed pitchfork bifurcation and clearly provides evidence that multiple equilibria exist when the lateral friction is small enough. The perturbed pitchfork arises because the mid-basin symmetry is obviously broken by the geometry and the wind forcing. Note that there is quite a range of Ekman numbers where two equilibria are (barotropically) stable.

The stationary solution (not shown) at location b in Figure 11(a) displays a Kuroshio path south of Japan with two recirculation gyres, and an extended jet near Japan similar to the elongated Kuroshio flow. A stable stationary solution on the upper branch (location c) is shown in Figure 11(b) for \( E = 1.5 \times 10^{-7} \), which is in the multiple equilibria regime. It displays a Kuroshio path south of Japan which is different from either the elongated or contracted states. Compared to the solution at location b, to which it is continuously

![Bifurcation diagram](image)

**Figure 11.** (a) Bifurcation diagram for the barotropic shallow-water model on the North Pacific domain with the Ekman number \( E = A_H/(2\Omega r_0^2) \) as control parameter. Drawn (dotted) branches indicate stable (unstable) steady states, whereas the Hopf bifurcation points are indicated by triangles. (b) Contour plot of sea-surface height deviations for the steady state solution at location c on the upper branch in (a). (c) Contour plot of SSH deviations for the steady state solution at location d on the upper branch in (a). The contour levels are scaled with respect to the maximum value of the field (from Schmeits and Dijkstra [21]).
connected, the anti-cyclonic recirculation gyre to the south of Japan has intensified and has caused the Kuroshio to deviate from the coast.

The second branch of solutions exists only for \( E < 1.8 \times 10^{-7} \), which is the position of the saddle-node bifurcation on this branch. The solution at \( E = 1.5 \times 10^{-7} \) (Figure 11c) displays a Kuroshio path south of Japan, with three recirculation gyres, and a large meander near Japan similar to the contracted state. The third steady state still at \( E = 1.5 \times 10^{-7} \) (location e) is unstable and the Kuroshio has different separation behaviour than for the other states. The important point from Figure 11 is that multiple equilibria are still present in the more realistic case and that the flow patterns differ in the separation and meandering behaviour of the Kuroshio, in particular in the number of recirculation gyres. The circulation patterns outside the WBC region are very similar for each solution.

In Pierini [22], the reduced-gravity shallow-water equations forced by a steady wind forcing derived from climatological winds are used to study the low-frequency KE variability. The governing equations of the model are:

\[
\begin{align*}
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + f \mathbf{k} \times \mathbf{u} &= -g' \nabla \tilde{h} + \frac{\tau}{\rho H} + A_H \nabla^2 \mathbf{u} - \gamma \mathbf{u} \mathbf{u} \\
\frac{\partial \tilde{h}}{\partial t} + \nabla (H \mathbf{u}) &= 0
\end{align*}
\]  

where \( \mathbf{k} = (0, 0, 1), \mathbf{r} = (\tau', 0, 0), \mathbf{u} = (u, v, 0) \) is the horizontal velocity vertically averaged in the upper active layer, \( \tilde{h} \) is the interface displacement (positive downwards), \( H = D + \tilde{h} \) is the upper layer thickness, where the undisturbed layer thickness is \( D = 500 \) m, the density of the upper layer is \( \rho = 1.0235 \) g cm\(^{-3}\) and in the reduced gravity the relative variation of density between the two layers is \( \Delta \rho / \rho = 0.0045 \). The reduced-gravity approximation is very appropriate (see Figure 5), and this value of \( g' \) fits well with the observations. Moreover, \( \gamma = 5 \times 10^{-4} \) m\(^{-1}\) parameterizes the interfacial friction. Finally, the spatial resolution and time steps are \( \Delta x = \Delta y = 20 \) km and \( \Delta t = 20 \) min, respectively. The domain of integration is shown in the upper panel of Figure 12 that includes the wind-stress curl map. The fundamental importance of the schematic coastline introduced in the western part of the ocean basin and the large zonal width of the latter is discussed by Pierini [68].

The model results yield a decadal chaotic self-sustained oscillation in substantial agreement with the altimeter observations of Qiu and Chen [48] (see the discussion below). A complex dynamical mechanism supporting this internal oscillation, and involving the bimodal behaviour of the Kuroshio south of Japan, was also proposed by Pierini ([22], see Section 4(a) therein). A first significant validation of the model results [22] is provided by the model-data comparison relative to the mean jet. The lower panel of Figure 12 shows the time averaged SSH field obtained by the model with, superimposed, the climatological surface dynamic height of Teague et al. [52]. The two pictures have the same \( y \) scale and the same \( x \) scale at latitudes near \( \varphi = 35^\circ \)N. The meandering of the simulated mean jet, its downstream structure, its intensity, and the northern and southern recirculation regions are in substantial agreement with the observed climatology. The point of detachment of the modelled Kuroshio is at \((35^\circ \text{N}, 141^\circ \text{E})\), in agreement with observations (note that the line of vanishing wind-stress curl is imposed at \( 37^\circ \)N, so the separation line of the WBCs represents a non-trival oceanic response to the winds, and clearly depends on the shape of the coastline). The first meander corresponds quite well with the observed one as far as its zonal width, curvature, location, and strength are.
concerned. The second meander is also in basic agreement with the observed one, although it appears shifted eastward by about one degree in longitude. East of the second crest the mean axis of the jet is slightly inclined southward in both the observed and modelled KE; a weaker meandering path about the axis of the jet is evident in both images as well, and the more intense spatial fluctuations present in the real ocean are usually accounted for by topographic interactions with the Shatsky Rise and, more eastward, with the Emperor Seamounts, all effects absent in this model. The recirculation gyre south of the main meanders appears also in basic agreement with the climatological one. Also the modelled path of the Kuroshio south of Japan shows similarities with the climatological WBC. Finally, another agreement worth emphasizing is the configuration of the subpolar and subtropical gyres. The northern cyclonic cell shows, in both observed and modelled data, a meridional SSH gradient that is larger than that found in the southern anticyclonic cell.

Figure 12. Upper panel: domain of integration and contour map of the curl of the wind stress (units in $10^{-8}$ N m$^{-1}$) used in the reduced-gravity shallow-water model [22]. Lower panel: time-averaged SSH with, superimposed (small panel), the climatological surface dynamic height map (dyn cm) relative to 1000 dbar of Teague et al. [52] (from Pierini [22]).
Let us now pass to consider the validation of the time-dependent flow. In Pierini et al. [23], model results for a value of $A_H = 220 \text{ m}^2 \text{s}^{-1}$ are compared with observations. In Figure 13, the annual-averaged SSH fields of [48] (colour images) are compared with the SSH fields obtained from model results (greyscale images) according to a synchronization of model/observations where the year 1993 is identified with model year 157 [23]. In 1993 the KE is in the elongated state and corresponding model solution ($t = 157$ yr) agrees well for the first large anticyclonic meander in position, shape and strength. After 11 years a similar situation is reached (year 2004, $t = 168$ yr), and the agreement is now even better for the first and second anticyclonic meanders and for the cyclonic meander south of Japan. For intermediate times, the variability found in the model results is in agreement with the observations. The disruption of the elongated state, accompanied by the disappearance of the two main anti-cyclonic meanders, occurs in less than 1 year (from year 1994, $t = 158$ yr to year 1995, $t = 159$ yr).

Also the observed KE path length $L_{KE}$ (Figure 6) and mean latitudinal position $\phi$ of the upstream KE axis (both defined in [48]) were compared with the corresponding model results for $A_H = 220 \text{ m}^2 \text{s}^{-1}$ in [23], as shown in Figure 14. The behaviour based on data can be summarized as follows (Figures 14a,b): (i) during the elongated state (roughly covering the periods 1993–1995 and 2002–2005) $L_{KE}$ and $\phi$ are both weakly varying about their mean value; (ii) during the recharging (transition) phase of the relaxation oscillation, that lasts about 7 years and connects those two periods, $L_{KE}$ yields a much larger (high frequency) variability while $\phi$ shows a clear positive trend until approximately the end of the transition. The model data for two successive cycles (Figures 14c,d and 8e,f) show the same qualitative behaviour and also an acceptable quantitative agreement both as far as the timing and the amplitudes are concerned.

Figure 13. Panels in colour: maps of yearly averaged SSH fields computed from altimeter data (adapted from Figure 2 of Qiu and Chen [48]. Panels in greyscale: snapshots of the SSH fields computed from the model [22] at the beginning of each year. A correspondence of year 1993 with model year 157 is made (from Pierini et al. [23]).
The bifurcation behaviour of the model solutions with $A_H$ as control parameter was preliminary investigated in [22] and more thoroughly considered in [23]. Figure 15 shows a bifurcation diagram obtained in [23] by performing a large number of forward time integrations. For each value of $A_H$ the curves with labels min and max give the range within which $E_A$ (the kinetic energy per unit mass integrated in sector A of Figure 12) varies after spinup. For $A_H < 230$ m$^2$s$^{-1}$ the two pairs of curves $b_{min} - b_{max}$ and $c_{min} - c_{max}$ are associated with the spontaneous switching of the trajectory between two equilibrium flows. It was found that in the interval (between the two dashed lines) $A_H = 240$ m$^2$s$^{-1}$ down to $A_H = 235$ m$^2$s$^{-1}$, the system undergoes an impressive change in behaviour. There is a transition from weak amplitude irregular oscillations at $A_H = 240$ m$^2$s$^{-1}$ (corresponding to gyre-mode variability) to a much larger amplitude relaxation-type oscillation at $A_H = 235$ m$^2$s$^{-1}$. In Figure 16(a), the abrupt transition from a small amplitude oscillation to a large amplitude relaxation oscillation is evidenced in the time series. In the $E_A - E_B$ phase plane (Figure 16b), where $E_B$ is the kinetic energy of the flow over a region B south of Japan, the view is even more dramatic; for $A_H = 240$ m$^2$s$^{-1}$ (blue curve) the trajectory occupies a relatively small area in state space while for $A_H = 235$ m$^2$s$^{-1}$ the trajectory suddenly explores a new high energy area in state space. This change in behaviour can be illustrated in more detail by considering the Probability Density Function (PDF) in the $E_A - E_B$ phase plane. Figure 16(c) shows that the PDF is confined in a restricted region around the unstable fixed point for $A_H = 240$ m$^2$s$^{-1}$, and then expands for $A_H = 235$ m$^2$s$^{-1}$, embracing regions well beyond the original basin of attraction, characteristic of a homoclinic bifurcation.
The important point of this subsection is that low-frequency KE variability, resembling observations both qualitatively and quantitatively, is found in a reduced-gravity shallow-water model under steady forcing. This variability very likely results from a homoclinic transition similar to those found in the quasi-geostrophic models of the double-gyre circulation (Section 3.3).

3.5 High-resolution ocean general circulation models

More sophisticated high-resolution (eddy-resolving) OGCMs have also been used to study the KE low-frequency variability, and to investigate its intrinsic component. Taguchi et al. [69] performed a multi-decadal (1950–2003) hindcast with a high-resolution (0.1°), eddy-resolving, global OGCM for the Earth Simulator (OFES), and found the low-frequency SSH variability in the North Pacific to be high near the KE front. By decomposing the SSH into Empirical Orthogonal Functions (EOFs), Taguchi et al. [69] found that the same variability is explained by two modes with meridional structures tightly trapped along the KE front. Both the spatial structure of the EOFs and time series of the associated Principal Components are in close agreement with observations (but direct comparison of the modelled KE paths with data such as the one shown in Figure 13, was not presented). A linear Rossby wave model like Equation (1) forced by observed winds successfully reproduced the time series of the leading OFES modes but failed to explain why their meridional structure is concentrated on the KE front, and is inconsistent
with the broad-scale wind forcing. The intrinsic variability was investigated by forcing the OFES model with a climatological forcing: this run reproduced an amount of variability comparable with that obtained in the hindcast run. Another conclusion was that, as the first two EOFs of the SSH anomalies of the hindcast simulation were found to be approximately a sum of the frontal-scale EOF and the EOF associated with the Rossby wave, the Rossby waves were suggested to ‘trigger’ the internal variability (see Section 5.3 for further insight into this important aspect).

Other high-resolution (eddy-resolving) model studies of the KE variability are those of Taguchi et al. [53], Nonaka et al. [70,71], Douglass et al. [40], Kurogi et al. [41], and Wang et al. [24]. Taguchi et al. [53] pointed to the role played by mesoscale eddies and related feedbacks in the KE dynamics. Nonaka et al. [70] used the Modular Ocean Model (MOM3) in an eddy-resolving configuration: low-frequency KE variability was found with both hindcast and climatological runs. Decadal SSH variability in the KE frontal zone was largely explained by propagation of baroclinic Rossby waves forced by anomalous Ekman pumping in the central North Pacific. This process alone could not, however, fully explain the corresponding variability in the subarctic frontal zone, where eastward propagating SSH anomalies off the Japanese coast seemed to be superimposed on the Rossby wave signals.

In general, although high-resolution models of the KE put in evidence of the ability of the ocean system to produce important intrinsic low-frequency variability, it is somewhat surprising that they do not produce a correct KE bimodality (e.g., [24,40,41]) or produce the bimodality but not the correct timing (e.g., [53]), while simpler models like [22,23] do. In Section 5.3 this crucial issue will be dealt with.
4. Predictability of the Kuroshio low-frequency variability

4.1 Precursors and error growth

In predictability studies of weather and climate, one important aspect is to explore the initial perturbation that most easily results in the occurrence of some weather or climate event. This initial perturbation is known as the optimal precursor (OPR) of the weather or climate event. The OPRs of El Niño/Southern Oscillation (ENSO) events have been explored using singular vector methodology by Thompson [72] and the Conditional Nonlinear Optimal Perturbation (CNOP) technique by Mu et al. [27]. Jiang and Wang [28] also used the CNOP approach to investigate the OPRs of atmospheric blocking.

Another important aspect in predictability studies is to identify the initial perturbation which leads to the largest prediction uncertainties under given initial constraint conditions when forecasting some weather or climate event. This initial perturbation is called the optimally growing initial error (OGIE). It is of importance to recognize the OGIE for forecasting the weather or climate event, which will deepen our understanding for prediction uncertainties and increase the skill of the prediction. Molteni et al. [29] and Toth and Kalnay [30] used different kinds of OGIEs as the initial perturbations in ensemble weather forecast systems to increase the forecast skill. Tang et al. [73] and Mu et al. [27] utilized the singular vector method and the CNOP approach, respectively, to investigate the effects of OGIEs on the predictability of ENSO.

As the OPRs and OGIEs are two different kinds of initial perturbations, the following question naturally arises: are the spatial patterns of the OPRs and OGIEs similar? Mu and Jiang [74] provided a positive answer to this question in a predictability study of atmospheric blocking. As they pointed out, it is very meaningful to address this issue, because it will help to identify the areas where targeted observations will improve the forecasts. If the spatial structures of OPRs and OGIEs for the Kuroshio path variability problem are localized and similar, targeted observations implemented over the sensitive area (e.g., defined by the patterns of OGIEs) will not only be able to reduce the probability of the appearance of the optimally growing initial errors in the prediction of the LM path, but also can help to capture the optimal precursor of the Kuroshio LM. These observations will then further increase the ability to forecast the Kuroshio LM path. On the other hand, the cost for ocean observation may be significantly reduced because one only needs additional observations over several localized areas. The determination of the sensitive areas according to the spatial structures of OPRs and OGIEs is therefore a key problem for targeted observation.

In the next section we report the results of predictability studies [24] on the development of the Kuroshio LM, in which the OPRs and OGIEs computed with the CNOP method are analysed in connection with targeted observations. To study nonlinear mechanisms of perturbation amplification, Mu [75] proposed the concepts of nonlinear singular vectors and nonlinear singular values which were then applied by Mu and Wang [76] to shallow-water flows (see also [77]). In general, one assumes that the equations governing the evolution of perturbations can be written as:

\[
\begin{align*}
\frac{\partial \mathbf{x}}{\partial t} + F(\mathbf{x}; \mathbf{x}) &= 0 \\
\mathbf{x}\big|_{t=0} &= \mathbf{x}_0
\end{align*}
\]

in \(\Omega \times [0, t_e]\) (6)

where \(\mathbf{x}(t)(x_1(t), x_2(t), \ldots, x_n(t))\) is the perturbation state vector and \(F\) is a nonlinear differentiable operator. Furthermore, \(\mathbf{x}_0\) is the initial perturbation, \(\mathbf{x}(t)\) is the basic state on
which the perturbation is superimposed, \((x, t) \in \Omega \times [0, t_e]\) with \(\Omega\) a domain in \(\mathcal{R}^n\), and \(t_e < + \infty\).

Suppose the initial value problem (Equation 6) is well-posed and the nonlinear propagator \(\mathcal{M}\) is defined as the evolution operator of Equation (6) which determines a trajectory from the initial time \(t = 0\) to time \(t_e\). Hence, for fixed \(t_e > 0\), the solution

\[ x(t_e) = \mathcal{M}(x_0; \bar{x})(t_e). \]  

(7)

is well-defined.

For a chosen norm \(\| \cdot \|\), the perturbation \(x_0\) is called the CNOP with constraint condition \(\|x_0\| \leq \delta\), if and only if

\[ J(x_0) = \max_{\|x_0\| \leq \delta} J(x_0), \]  

(8)

where the ‘cost function’ \(J\) is given by

\[ J(x_0) = \|\mathcal{M}(x_0; \bar{x})(t_e)\|. \]  

(9)

The CNOP is therefore the initial perturbation whose nonlinear evolution attains the maximal value of the functional \(J\) at time \(t_e\) with the constraint conditions, and can thus be regarded as the most (nonlinearly) unstable initial perturbation superimposed on the basic state.

4.2 Predictability of the Kuroshio Large Meander

Recently, several studies have been carried out to understand the predictability of the Kuroshio LM. Some of them tried to improve the prediction of the LM state by using different initialization techniques and then estimating the predictability horizon of the Kuroshio LM event [78–81]. Other studies examined the effects of initial errors on the prediction of the LM. For example, Ishikawa et al. [82] used a reduced-gravity shallow-water model and adjoint sensitivity method to investigate the sensitivity of the predictive results of the LM state to initial errors. Fujii [83] utilized a singular vector approach to explore the influences of initial errors on the prediction of the LM state.

In the remainder of the section we will focus on the predictability of the Kuroshio LM based on the CNOP method described in Section 4.1. Wang et al.’s [24] model is governed by the reduced-gravity shallow-water equations implemented in the North Pacific with an eddy permitting resolution. The numerical setup is similar to the one used in Pierini [22] described in detail in Section 3.4. The main differences are the use of spherical coordinates and a value of the lateral eddy viscosity \(\nu_e = 450 \text{ m}^2\text{s}^{-1}\) which is larger than that of the reference simulation in [22]. The forcing is provided by a monthly climatological wind stress, so, as in [22], the cause of the interannual variability produced by the model is not the wind but nonlinear intrinsic oceanic mechanisms, as described in Section 2.2. The model is integrated for 40 years and the results of the last 31 years are analysed. The Kuroshio path index is used as a measure of the state of the Kuroshio path and is defined here as the latitude of the southernmost point of the Kuroshio axis within the longitude band 136°E to 140°E, where the Kuroshio axis is represented as the 520 m contour of upper layer thickness. The Kuroshio takes a LM path when the index is small, otherwise it takes a NLM path. Figure 17(a) shows the time series of the modelled...
Kuroshio path index. It is found that the Kuroshio path displays strong interannual variability, and the LM path occurs 6 times in the 31 model years. Wang et al. [37] investigated the transition processes between the NLM and LM paths in this model: the results indicate that the growth processes of the meander in the transition from NLM to LM path are adequately captured, although the lifetimes of the modelled LM paths are relatively short.

With $\mathbf{x}$ denoting the perturbation state vector of the shallow-water model, namely $\mathbf{x} = (u, v, h)$, the model can be formally written as in Equation (7), where $\mathbf{x}_0 = (u_0, v_0, h_0)$ indicates the initial perturbation, $M$ is the nonlinear propagator and $\mathbf{x}(t) = (u_t, v_t, h_t)$ is the state vector at time $t$. To investigate the transition from NLM path to LM path with the CNOP approach, the three time-dependent states $\mathbf{x}$ with a NLM path identified by the dashed boxes in Figure 17(a) are used in [24] as reference trajectories (Figure 17(b) shows a schematic diagram describing the mathematical process). The initial perturbations that cause those NLM-LM transitions are then explored. In order to find

![Figure 17](image.png)

**Figure 17.** (a) Time series of the Kuroshio path index calculated from the model results [24]. The time-dependent states corresponding to the Kuroshio path indices in the dashed boxes are utilized as reference trajectories. (b) Schematic diagram of calculating OPR and OGIE. The initial reference state determines a reference trajectory taking the NLM path when integrating the nonlinear model (thick line). Based on this reference trajectory, the OPR is calculated. When the OPR is superimposed on the initial reference state, the new initial state defines a trajectory which results in the onset of the LM path (thin line). This new trajectory is regarded as the reference one for computing the OGIE (adapted from Wang et al. [24]).
such perturbations, a nonlinear constraint optimization problem is defined by Equation (8), in which the constraint norm is defined as the total energy of the initial perturbation in the whole model domain. The cost function (Equation 9) is defined by the squared Eulerian norm of the kinetic energy of the initial perturbation in the Kuroshio region south of Japan, where the Kuroshio LM path occurs. This norm is chosen because it is able to clearly distinguish different Kuroshio path states. The initial perturbations thus obtained result in transitions from a NLM path to a LM path and are called OPRs of the Kuroshio LM path: they are the initial perturbations which most easily result in the formation of the Kuroshio LM path.

Initial errors may also have important impacts on the prediction of the LM path. In order to examine these effects, Wang et al. [24] used the formation process of the Kuroshio LM path caused by the OPR as a new reference trajectory to explore the OGIE (Figure 17b). Thus, a new optimization problem is defined to seek the OGIE that satisfies a given constraint condition of initial error. The norms for constraint condition and cost function are the same as those used when calculating the OPRs. The solution of the corresponding Equation (8) has the maximum nonlinear growth and thus is called the OGIE. From the formulation of the associated optimization problems, it is found that the main differences between the OPR and OGIE are their different reference trajectories. As shown in Figure 17(b), the reference trajectory for the OPR is the NLM path (thick line) and the LM path has not occurred. The OPR causes the NLM path to transform into the LM path. For the OGIE, the reference trajectory is the process of the transition from NLM to LM path caused by the OPR (thin line). In this case, the LM path has occurred. The OGIE results in a new trajectory (dashed line) that departs from that caused by the OPR (thin line). The departure represents the forecast error with respect to the process of the onset of the LM path (caused by the OPR).

Figures 18(a–c) show the OPRs (shaded) for each of the three cases shown by the dashed boxes in Figure 17(a) (the corresponding PV distribution is given by the contours).

Figure 18. The upper-layer thickness components (shaded) of OPRs in the southern region of Japan and potential vorticity distribution (contour) of initial reference state for (a) Case 1, (b) Case 2, and (c) Case 3 (see Figure 17a). Unit for upper layer thickness is m, and contour interval for potential vorticity field is $1.1 \times 10^{-8}$ s$^{-1}$ m$^{-1}$. (d–f): as in (a–c) but for the OGIEs (adapted from Wang et al. [24]).
We can see from the figure that although some differences exist in the details of spatial patterns of the upper layer thickness components of OPRs for the three cases, the patterns are all localized, and their large amplitudes are mainly located at the southeast of Kyushu (marked in Figure 18a). In order to investigate the effects of the OPRs on the Kuroshio path, the nonlinear model is integrated with the OPRs superimposed on the initial reference states. Figures 19(a–c) show the Kuroshio paths caused by the OPRs and those in the reference states at day 450. As expected, the OPRs induce the transitions from NLM path to LM path in all three cases and finally result in the formation of typical LM paths. The OGIEs are then calculated for each case based on the new reference trajectory corresponding to the transition process from NLM path to LM path caused by the OPRs. Figures 18(d–e) show the upper-layer thickness components of the OGIEs and the PV distributions of the new initial reference states for the three cases. Their large amplitudes are localized near the southeast of Kyushu. Similar to Figures 18(a–c), Figures 18(d–e) also illustrate that there are strong positive PV gradients in the southeast of Kyushu, which, in addition, also change sign in this region (not shown). This may induce barotropic instability, which results in a fast growth of the initial errors. Figures 19(d–f) show the Kuroshio axes (solid lines) in the new reference states at day 450 and those (dashed lines) caused by OGIEs for the three cases. For Case 1 and 3, the amplitudes of forecasted LM paths caused by OGIEs are weaker than those of the reference states. Quantitatively, for Case 1, the Kuroshio path index (see above for its definition) is 31.4 for the forecasted path and 29.6 for the reference state. For Case 3, the path index is 30.8 for the forecasted path and 28.8 for the reference states. For Case 2 (Figure 19e), the results are opposite:
the amplitude of the forecasted LM path induced by OGIE is stronger than that of the reference state. Correspondingly, the Kuroshio path index is 28.6 for the forecasted path and 30.0 for the reference state.

We conclude this section by considering the application of the preceding analysis to targeted observations. Targeted observation studies have been carried out for more than 10 years in meteorology and viewed as an effective way to increase the ability to predict weather numerically[84–86]. In recent years, some researchers [26,87,88] have paid attention to this study in physical oceanography (in Section 5.2 the study [26] will be considered in detail). A key problem of targeted observations is how to determine the sensitive area in which the additional observations are assimilated to obtain better forecast results than observations taken in other areas. The localizations and similarities of the spatial structures of the OPRs and OGIEs discussed above suggest defining the sensitive area by the spatial structures of the OGIEs obtained using the CNOP method. In this way, one can not only decrease the probability of the appearance of the OGIEs, but also capture the precursor signal of the OPR. This is expected to improve the prediction of the Kuroshio LM path.

Figure 20(a) shows the total energy distribution of the OGIE for Case 1. It is found that the large amplitude region is located mainly to the southeast of Kyushu. Figure 20(b) illustrates the total energy distribution of the OPR: as expected, its large amplitude region is almost the same as that of the OGIE. The sensitive area is defined as a box, large enough to contain the large amplitude of total energy and is denoted as R3 in Figure 20(a). From the distributions of PV (Figure 18), one can see that the sensitive area corresponds to the region of strong PV gradients. To further examine whether the forecast is more sensitive to the initial errors in the area R3 than other areas, the evolutions of random initial errors in different local areas are investigated. Figure 20(a) shows nine local regions of the same size, including the sensitive region R3. These regions mainly cover the area south of Japan and upstream of the KE, where the initial perturbations may have relatively great effects on Kuroshio path variations. In each region, 40 random initial error fields are generated, and the kinetic energies of the 40 forecast errors thus induced are calculated. Their average value is listed in Table 1, which illustrates that the largest forecast error is the one in the sensitive area R3 identified by the spatial structure of OGIE. This reveals that the prediction of the Kuroshio LM path is more sensitive to the initial error in the sensitive region R3 than in other randomly selected areas. These results suggest that the growth of initial error strongly depends on its location, and the sensitive area determined by the spatial structure of OGIE is effective, in which the initial error will acquire
rapid growth. It is also interesting to note that the forecast error caused by the part of Ogie obtained by the CNOP approach is much larger than that induced by random error. This implies that in the sensitive area, the spatial patterns of initial errors have key influences on the growth of forecast errors.

In conclusion, one can infer from the above results that if the targeted observations are implemented in the sensitive area determined by spatial structures of OGIEs, the initial condition in this region can be improved, and hence the forecast error will be significantly reduced. Simultaneously, the observations implemented in the sensitive area can also capture the precursor signal of the occurrence of the Kurashio LM path because of the similarities between the spatial patterns of the OPR and OGIEs. All of these illustrate that targeted observations may greatly improve the prediction of the Kuroshio LM path.

5. Predictability of the Kuroshio Extension low-frequency variability

5.1 Lyapunov exponents

Very useful information concerning the sensitivity of flow trajectories of the shallow-water model [22] to initial conditions (and hence, concerning the degree of predictability of the flow) is provided by the rate of divergence of initially nearby points. Pierini et al. [23] investigated this by computing a field of finite-size Lyapunov exponents (FSLE, e.g., Aurell et al. [89]). Let $X_i$ be a trajectory sampled at times $t_k = kdt$ in the $E_A - E_B$ plane (see Section 3.4), and let $\delta |_{t=0} = \delta_0$ be the initial distance (e.g., according to the Eulerian metrics) between two trajectories. The corresponding Lyapunov exponent $\lambda$ defined in

$$\delta |_{t} = \delta_0 e^{\lambda t}$$

is a parameter characterizing the evolution of those trajectories: the case $\lambda > 0$ is indicative of extreme dependence on initial conditions and, therefore, of chaotic dynamics. The so-called doubling time Lyapunov exponent (DTLE, a particular case of FSLE) is defined as $\lambda_2 = \ln 2 / \tau_2$, where $\tau_2$ is the time (if it exists) after which the distance becomes $\delta |_{t=\tau_2} = 2\delta_0$. 

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Table 1. Mean of the kinetic energies of the forecast errors caused by 40 random initial errors in each of the nine regions of Figure 20(a) (from Wang et al. [24]).

<table>
<thead>
<tr>
<th>Labels</th>
<th>Regions</th>
<th>Mean of Kinetic Energy ($\times 10^9 m^5 s^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>(31°N–33.64°N; 136°E–140°E)</td>
<td>9.6</td>
</tr>
<tr>
<td>R2</td>
<td>(32.2°N–34.6°N; 140°E–144°E)</td>
<td>3.1</td>
</tr>
<tr>
<td>R3</td>
<td>(29.8°N–32.2°N; 132°E–136°E)</td>
<td>26</td>
</tr>
<tr>
<td>R4</td>
<td>(29.8°N–32.2°N; 136°E–140°E)</td>
<td>9.6</td>
</tr>
<tr>
<td>R5</td>
<td>(29.8°N–32.2°N; 140°E–144°E)</td>
<td>4.7</td>
</tr>
<tr>
<td>R6</td>
<td>(28.6°N–31°N; 128°E–132°E)</td>
<td>1.7</td>
</tr>
<tr>
<td>R7</td>
<td>(28.6°N–31°N; 132°E–136°E)</td>
<td>7.6</td>
</tr>
<tr>
<td>R8</td>
<td>(28.6°N–31°N; 136°E–140°E)</td>
<td>6.5</td>
</tr>
<tr>
<td>R9</td>
<td>(28.6°N–31°N; 140°E–144°E)</td>
<td>2.7</td>
</tr>
</tbody>
</table>
Pierini et al. [23] computed $\lambda_2$ as follows. The $E_A - E_B$ plane was divided into $n \times m$ rectangular cells $\sigma_{ij}$, each of area $d\sigma$. The DTLE,

$$\lambda_2(i,j) = \frac{\ln 2}{\langle \tau_2 \rangle_{ij}}$$  \hspace{1cm} (11)

was then evaluated by averaging $\tau_2$ in each cell over a large number of couples of initial conditions obtained from a single trajectory. Moreover, in order to characterize a trajectory $X_k$ in terms of the degree of predictability of the state space regions it encounters, [23] proposed the use of an extended form of Lagrangian Lyapunov exponent following $X_k$, (see Crisanti et al. [90] for the classical definition of the parameter), defined as

$$\Lambda_X(t_k) = \sum_{ij} \lambda_2(i,j) q_{ij}(X_k)$$  \hspace{1cm} (12)

where the parameter

$$q_{ij}(X_k) = \begin{cases} 
1 & \text{if } X_k \in \sigma_{ij} \\
0 & \text{if } X_k \notin \sigma_{ij}
\end{cases}$$

localizes the trajectory in the $E_A - E_B$ plane at time $t_k$. It should be borne in mind that $\lambda_2$ and $\Lambda_X$ are computed on the $E_A - E_B$ phase plane, so the predictability properties implied by such Lyapunov exponents concern only the energy of the system, not its actual state. In principle, cases might exist in which the evolution of the energy of different spatial sectors of the system could be very predictable and, at the same time, the flow evolution could be very sensitive to the initial conditions (see the discussion of Figure 22 below), but the contrary would not be allowed. In practice, since the bimodality of the KE is described quite well by $E_A$ and $E_B$, the quantities $\lambda_2$ and $\Lambda_X$ are expected to provide significant information about the possibility of predicting the evolution of the KE state, at least in its gross features.

Figure 21 shows $\lambda_2$ for $A_H = 220$ m$^2$s$^{-1}$, computed from 400-yr-long time series. In the region $\Pi_m$, $\lambda_2$ is very small everywhere: this implies a slow divergence of trajectories and, consequently, a relatively high degree of predictability (in the sense stated above). On the other hand, it is in the reconnection regions $\Pi_{rec1}$ and $\Pi_{rec2}$ that $\lambda_2$ assumes very large values, implying a highly unpredictable state. Figure 22(c) shows $\Lambda_X$ for the reference period $t = 145 - 169$ yr (corresponding to the two bimodal cycles discussed in Section 3.4), where $E_A$, $E_B$ (Figure 22a), $L_{KE}$ and $\bar{\phi}$ (Figure 22b) are also shown for the sake of comparison. Three main stages (S1, S2, and S3) of the relaxation oscillations can be identified.

In stage S1, during the intervals $t \approx 147 - 151$ yr and $t \approx 159.5 - 165$ yr (for the first and second cycles, respectively) $\Lambda_X$ is very small, so $E_A$ and $E_B$ yield the highest predictability. Here we are in the first half of the transition phase, that is, just after the disruption of the large-meander phase and, subsequently, in the recharging phase of the relaxation oscillation (Figure 22(a), thick line) when the KE patterns are more variable (Figure 13) and more convoluted (Figure 22(b), thick line) and $\bar{\phi}$ is increasing (Figure 22(b), thin line). In other terms, the energy evolution proves less chaotic when the variability and convolution of flow patterns are maximum (this occurs in region $\Pi_m$ of state space). Such property is somewhat unexpected, but is only apparently paradoxical and appears to be
very distinctive of the KE theory [22,23]. The concomitant evolution of $E_A$ and $E_B$ in this stage of the oscillation differs little from cycle to cycle, but this hides the existence of highly variable mesoscale features (e.g., as shown by $L_{KE}$) that feed the KE jet and that, at the same time, are presumably very unpredictable.

In stage S2, during the intervals $t \approx 151 - 154 \text{ yr}$ and $t \approx 165 - 168 \text{ yr}$ (for the first and second cycles, respectively) $\Lambda_X$ is highly variable, with peaks an order of magnitude larger than in the previous phase, so $E_A$ and $E_B$ yield a highly chaotic behaviour and are, therefore, highly unpredictable ($\lambda_2 = 100 \text{ yr}^{-1}$ corresponds to a doubling time of just $\tau_2 \approx 2.5 \text{ days}$). This occurs when the KE is reaching its maximum energy, the large-meander state is attained, $\phi$ is at its maximum, and $L_{KE}$ has a weak variability. Moreover, the Kuroshio south of Japan yields a strong variability (Figure 22(a), thin line) that, according to [22,68], is associated with the ejection of cyclonic mesoscale eddies detaching from a meridionally elongated cyclonic meander (a process well known to occur south of Japan, e.g., [14]). So, it is basically in connection with this process (confined outside the KE region and occurring mainly in $\Pi_{rec2}$ and in part of $\Pi_{rec1}$) that the large values of $\Lambda_X$ emerge.
In stage S3, during the interval $t/C_2 \approx 154$ yr (referring to the first cycle; the corresponding interval for the second cycle is not included in the graph), $\Lambda_X$ is less variable than in stage S2 but with several very large peaks (corresponding to $t_2/C_2 \approx 1$ day), implying a very unpredictable system. This occurs mainly in sector $\Pi_a$ (Figure 21) when the KE is in the large-meander–elongated mode but is weakening, not being fed by the flux of negative relative vorticity coming from the south, which is dissipated within the region of the southern cyclonic meander (see [22], Section 4a therein).

In conclusion, the concomitant evolution of $E_A$ and $E_B$ is less chaotic when the KE jet is weak, more variable and more convoluted, and in its recharging phase (stage S1). It is very chaotic when the KE jet is in its large-meadner phase (stage S2), and also when it weakens, as long as it is in the large-meadner state (stage S3), in this last case presenting distinct episodes of very large DTLE. These conclusions provide valuable insight into the predictability of the low-frequency variability of the KE system.

### 5.2 Data assimilation

Our direct view of the oceans and atmosphere comes from remote sensing measurements and from still relatively sparse observations. Data assimilation combines the information gained from observations with computer simulations to obtain a three-dimensional representation of the current state of the ocean and atmosphere. An accurate estimate of the current state is particularly important for forecasting the future evolution. Because of the chaotic nature of these dynamical systems, uncertainties present in the initial state grow
rapidly, reducing the utility of the forecast. In this context, Kramer et al. [26] described an ensemble-based method for measuring the effect of observations on the KE decadal transitions: using model simulations from [22] and SSH observations, they applied the method to determine the best measurement locations to monitor the KE system for its optimal prediction.

The general idea is to perform an ensemble model forecast with the reduced-gravity shallow-water model of Pierini [22]. We have just seen in Section 5.1 that the corresponding system has positive Lyapunov exponents, so its chaotic nature limits the prediction time, as initial perturbations grow over time. The uncertainty in the initial state is thus assigned to the PDF $p(X_0)$; the ensemble samples the time averaged probability distribution of the system state. Observations are then used to update the ensemble using a particle-filtering technique (see Van Leeuwen [31] for an overview concerning geophysical systems). For this approach, neither the assumption of Gaussian error statistics nor the linearization of the model is required.

The evolution of the initial KE state $X_0$ is provided by the state vector $X_k$ given by

$$X_{k+1} = M_k(X_k, \zeta_k)$$

where $M_k$ is the system’s propagator associated with model [22] and $\zeta_k$ is a zero mean, white-noise sequence. Observations $Y_k$ become available at discrete times: the aim of a data-assimilation technique is to obtain a conditional probability density function $p(X_k | Y_{1:k})$ of the state at $t = t_k$. Taking a Monte Carlo approach, $p(X_k)$ is randomly sampled by a weighted ensemble of $N$ model realizations $X_k^i$ (also called particles),

$$p_N(X_k) = \sum_{i=1}^{N} w_k^i \delta(X_k - X_k^i)$$

with weights $w_k^i$ ($\delta$ is the Dirac delta function). The initial state of each ensemble member is uniformly drawn ($w_0^i = 1/N$) from the initial PDF $p(X_0)$. In a particle-filtering method, $p_N(X_k | Y_{1:k})$ is obtained recursively in a prediction step and an update step. Assume that $p_N(X_{k-1} | Y_{1:k-1})$ is known, that is, the particle states $X_{k-1}^i$ and the weights $w_{k-1}^i$ are known. Now, the next observation $Y_k$ becomes available: it can be shown that the new weights to be inserted in Equation (14) are then given by:

$$w_k^i = \frac{p(Y_k | X_k^i)}{p(Y_k)} w_{k-1}^i$$

The particular particle filter method is known as Sequential Importance Sampling (e.g., Doucet et al. [91]).

The application of the particle filter method allows one to sample the non-Gaussian probability distribution of the state vector. To specify the amount of uncertainty in the probability density function, entropy-based measures are favourable. Kramer et al. [26] adopted two measures to quantify the predictability of a KE ensemble forecast. The first is the Predictive Power (PP) introduced by Schneider and Griffies [92], which is a measure of the uncertainty relative to the climatological variance. The second is the Potential Prediction Utility, as introduced by Kleeman [93], which additionally incorporates a signal-to-noise component in the measure (see also Xu [94]).
The PP is based on the entropy \( S_p(X) \), a measure for the uncertainty associated with the PDF \( p(X) \) of variable \( X \) (Shannon [95]). The differential entropy is defined as

\[
S_X = -\kappa \int p(X) \ln p(X) dX
\]  

(16)

Here, \( \kappa \) is a constant that determines the unit of entropy. The particle filtering allows one to obtain an approximation to the probability density function \( p_N(X | Y_{1:k}) \) of the state vector. With the PDF available, the entropy \( S_p(X) \) of the ensemble forecast can be calculated. Schneider and Griffies [92] defined the PP, indicated by \( \alpha_X \), of an ensemble forecast as:

\[
\alpha_X \equiv 1 - \exp[-S_q(X) + S_p(X)]
\]

(17)

The entropy \( S_q(X) \) is calculated from the PDF of the climatology \( q(X) \), and can be considered as the uncertainty when only the climatological mean is known.

Instead of forecasting the actual KE by assimilating real observations in the shallow-water model, an identical-twin experiment was performed in [26]. Here, one model realization is considered to be the ‘true’ evolution of the KE. Equivalent observations are produced by taking measurements from this synthetic truth and adding observational errors. This allows one to produce observations for different variables, like the SSH or the velocity field, with different error distributions. Then, an analysis is performed by assimilating the observations in an ensemble run of the shallow-water model using the particle-filtering technique. The advantage of the identical-twin experiment is that one is sure that the evolution of the true KE is captured by the ensemble run with the shallow-water model. A failure to describe the truth is then not caused by the model, but either by insufficient observations or by the inadequacy of the data-assimilation method. In [26] a particular setup for the ensemble is used: at each time, the ensemble samples the climatological PDF \( q(X) \) by drawing the initial conditions \( X_i^0 \) from the climatological distribution.

The predictability time of an ensemble prediction of the KE was investigated in [26] by using the PP and potential prediction utility with the shallow-water model under stochastic wind forcing. This is a predictability study of the second kind [96], where the influence of uncertain boundary conditions – in this case in the wind stress – on the predictability is determined. For this purpose, a 96-member ensemble that starts from identical initial conditions was run. The initial state \( X_i^0 \) is obtained from a 50-yr spinup model run. In Figure 23(a), the forecast for the 96 ensemble realizations of the stochastically forced shallow-water model starting from a single initial state is shown in terms of the KE energy \( E_A \) (we have seen in Sections 3.4 and 5.1 how this parameter is able to provide valuable information on both the KE bimodality and its predictability in terms of Lyapunov exponents). In Figure 23(b), the PP (Equation 17) of the ensemble is given for a time span of 40 yr (with no uncertainty present in the initial conditions and hence the ensemble starts with a predictive power equal to unity). The PP is calculated using \( p(E_A) \) instead of the PDF \( p(X) \) of the full state vector (so only the PP of the energy signal is investigated and not that of the complete information provided by the forecast). During the first 5 yr the PP remains high. Then, over a one-year period the PP drops to \( \alpha_E = 0.5 \). Subsequently, the PP rises and ranges between \( \alpha_E = 0.6 \) and \( 0.8 \) for years 6–13.

The optimal monitoring of the KE was investigated in [26] by analysing how the uncertainty in the PV field \( Q(x, t) = (\zeta + f)/H \) is decreased by assimilating observations.
PV is a conservative advective tracer only influenced by forcing and dissipation; distributions of PV contain important information about the flow dynamics. To investigate the uncertainty in the PV, the PP (Equation 17) now becomes:

$$a_Q(x,t) = \exp\left[-S_q(Q)(x,t) + S_p(Q)(x,t)\right]$$  

(18)

The climatological entropy of the PV at point $x$, $S_q(Q)(x,t)$, is calculated using the $Q'(x,t)$ from the unweighted ensemble members. The study of $a_Q(x,t)$ reveals that there are differences in the performance of the moorings, depending on the state of the KE. Moreover, optimal observation locations are found to correspond to regions with strong PV gradients. Such PV gradients are a condition for barotropic instabilities to occur, which can lead to fast growth of perturbations. Measuring at these locations would prevent this potential growth of uncertainty. The optimal location is in the elongated jet during the high-energy state of the KE. This seems to confirm that during this state the KE is dynamically decoupled from the southern recirculation gyre, as suggested in [22].

The question of what is the single optimal location for reconstructing the decadal oscillations in the kinetic energy of the KE is answered in [26] by assimilating velocity and SSH observations at each and every grid point. Assimilating the observations for a given point yields a PP of $\overline{\alpha}_E$ (averaged over 40 synthetic truths). In Figures 24(a,b), this value is mapped at the location where the observations originate. The optimal measurement location is simply the one tagged by the highest predictive power. For an analysis based on velocity observations, the optimal location is at (33°N, 139°E).
assimilating SSH observations the optimal location is at (32°N, 139.5°E). For both velocity and SSH observations, the optimal location is in the region where the Kuroshio boundary current detaches from the coast and has a large meander. The regional experiments of the KE (WESTPAC, KERE, and KESS) were situated at different locations. In the context of the reduced-gravity model [22], the WESTPAC study at 152°E would be situated too far east to adequately capture the transitions of the KE.

Finally, the question remains if the optimal observation location is optimal for both the high-energy elongated state of the KE and for the low-energy contracted state. To check this, [26] selected the synthetic truths that correspond to one of the KE states and calculated the average PP. The result is presented in Figures 24(c,d) for both the low- and high-energy states. For the low-energy state, the optimal location coincides with the overall optimal location determined from Figures 24(a,b). However, during the high energy state the optimal location is at (36°N, 145.5°E), where the energetic jet starts to meander. Measuring velocity observations in the detached Kuroshio or in the first meander of the KE gives better PP than measuring SSH along the KE path during the elongated-jet state (see [26] for a detailed discussion).

5.3 Role of wind forcing

In this section we will discuss the role of wind variations in the predictability of the KE low-frequency variability. In Sections 3.2–3.5 model results are presented supporting the hypothesis that the observed KE bimodal cycle is basically sustained by intrinsic
nonlinear oceanic mechanisms. In this view the atmospheric variability does not play a dominant role in the KE evolution. On the other hand, we saw in Section 3.2 that wind stress anomalies in the eastern North Pacific generate baroclinic Rossby waves whose arrival in the KE region is in synchrony with the KE cycle (compare Figures 6 and 7, see [49] for an updated view of this behaviour): this suggests that wind variations do play a relevant role. We are therefore in the presence of a paradox opposing two apparently contrasting views (referred to as the intrinsically generated variability view and Rossby wave view by Pierini and Dijkstra [50]).

It is clear that these two views can be reconciled, and the paradox resolved, only if it is demonstrated that the KE bimodality is the manifestation of a nonlinear intrinsic ocean mode affected or even excited by Rossby wave trains generated by the wind in the central and eastern North Pacific. In this respect, based on their eddy-resolving OFES hindcasts, Taguchi et al. [69] suggested that the broad-scale Rossby waves generated by the basin-scale wind variability excite intrinsic modes of the KE jet, thus reorganizing the SSH variability in space. Qiu and Chen [49] and Taguchi et al. [53] share the same view, and point to the role played by mesoscale eddies and related feedbacks in the KE dynamics (see also [70,71]). An approach to the problem based also on the powerful concepts of dynamical systems theory was adopted by Pierini [97]: the KE relaxation oscillation [22] was shown to emerge either spontaneously (self-sustained oscillation) in a parameter range past a homoclinic bifurcation in state space (a so-called tipping point) or under an appropriate red noise wind forcing below that bifurcation according to the coherence resonance mechanism [98]. To study the same problem on a more conceptual level, a low-order ocean model was developed by Pierini [99]: the combined effect of coherence resonance and of a periodic forcing could then be analysed. The effect of a noise forcing on the critical transitions of this low-order KE model was then analysed in [100].

A model study specifically devoted to investigating the resolution of this paradox was developed by Pierini [32]; the model [22] is extended to include an additive time-dependent forcing representing schematically the North Pacific Oscillation (NPO; the second dominant mode in atmospheric sea level pressure known to drive the North Pacific Gyre Oscillation — NPGO, e.g., [101–103]). In a reference simulation the system is excitable (i.e., it is set below the critical conditions, hence the KE relaxation oscillation does not emerge spontaneously). The NPO forcing efficiently excites the oscillations, whose timing and teleconnection with NPO are found to be in good agreement with the altimeter data [49]. Figure 25(a) shows $L_{KE}$ for the reference simulation obtained under the same climatological forcing used in [22] but with the additional time-dependent NPO component shown in Figure 25(c). Figure 25(b) shows the teleconnection provided by the baroclinic Rossby waves. The emerged dynamical mechanism is explained as a case of intrinsic variability in an excitable dynamical system triggered, and therefore paced, by an external forcing.

After this model validation and an analysis confirming the robustness of the obtained ocean response, a sensitivity experiment, was carried out in [32] by varying the lateral eddy viscosity coefficient $A_H$. The emergence of the KE bimodality with a correct timing is found to be extremely sensitive to changes in this parameter; the implications of such sensitivity for deficiencies in the reproduction of the KE low-frequency variability in more realistic North Pacific OGCMs (e.g., [24,40,41,53,70] is then discussed. The conclusion drawn in [32] is that in excitable models of the North Pacific, tuning of dissipative effects (both parameterized and resolved) could lead to a better representation and phasing of the KE bimodality or, conversely, even to the disappearance of the latter, so that...
sensitivity to dissipation should accurately be assessed, otherwise fundamental aspects of the variability could be missed.

This delayed mechanism has important implications on the KE predictability. While the methods presented in Sections 5.1–5.2 allow the assessment of the predictability during the various stages of the KE chaotic decadal oscillation, the knowledge of the atmospheric variability in the central and eastern North Pacific may allow prediction of the

Figure 25. (a) KE path length $L_{KE}$ for the reference simulation obtained by Pierini [32] under the same climatological forcing used in [22] (Figure 12, upper panel) with the additional NPO component, whose time dependence is shown in panel (c). (b) Hovmöller diagram of the SSH anomaly at $35^\circ$N showing the baroclinic Rossby wave field providing the teleconnection mechanism (adapted from Pierini [32]).
arrival of the negative SSH anomaly carried by Rossby waves in the KE region, which in turn excites the recharging phase of the KE relaxation oscillation, as occurred in 1995 and 2006 (see Figures 6 and 7; e.g., see [71]). In this simplified framework the KE state is determined by the past wind stress, with the ocean playing an active role in the KE dynamics but a passive role with respect to the atmosphere. On the other hand, the strong air-sea heat exchanges taking place along WBC paths (e.g., [5,104]) lead to feedbacks and to a two-way ocean-atmosphere coupling that may affect the KE predictability. In a recent study based on altimeter observations and OFES outputs, Qiu et al. [105] suggested that taking into account the forcing by the KE feedback-induced surface wind anomalies can improve the long-term prediction of the dynamic state of the KE system. A SSH-based KE index from 1977 to 2012 is shown to be predictable at lead times of up to ~6 yr as a result of two dynamic processes, namely, the oceanic adjustment described above and the influences of the low-frequency KE variability on the extratropical storm tracks and surface wind stress curl field across the North Pacific basin. The prediction is referred to that specific KE index: it might be argued that this may not identify unequivocally the KE bimodality in OFES (which, as a matter of fact, was not demonstrated unambiguously, e.g., through the time series of $L_{KE}$ and/or the yearly-averaged SSH maps such as those shown in Figures 13 and 14). In any case, studies like [105] open new promising perspectives for assessing the predictability of the KE low-frequency variability.

Finally, OGCM results with different wind forcing have been used by Penduff et al. [106] to provide — for the first time — a global mapping of the intrinsic oceanic low-frequency variability. The Nucleus for European Modelling of the Ocean (NEMO) code (Madec et al. [107]) with $1/4^\circ$ resolution driven only by the climatological annual cycle reveals that the intrinsic SSH low-frequency variability thus obtained (denoted $I(i)$) explains $50-55\%$ of the variance of its fully forced counterpart (denoted $T(i)$) in eddy-active regions such as WBC systems (including the Kuroshio system). In view of the above discussion, the assumption $T = I + F$ adopted in [106] (where $F(i)$ is the forced component) appears to be not fully justified because $I$ is only part of the total intrinsic variability (that here we call $I_F$). In fact, one can write $I_F = I + I'$, where $I'$ is the intrinsic variability triggered by low-frequency (interannual) changes of the atmospheric forcing, with $I$ having a chaotic character and $I'$ yielding some form of synchronization with the atmosphere (the two KE cycles obtained by Pierini [32] would represent $I'$, with $I \equiv 0$). This may explain why Penduff et al. [106] did not find significant temporal correlation between $I$ and the AVISO altimeter-derived low-frequency variability. In general, a strong model sensitivity of $I$ and $I'$ on the representation of dissipative processes can be expected [32], which makes it problematic to simulate realistic amplitudes and spatial patterns of $I_F$ in high-resolution OGCMs. The horizontal spatial resolution also plays an essential role in the intrinsic response [106,108]. A more realistic modelling of the global intrinsic oceanic low-frequency variability (as started by Penduff et al. [106]) will therefore be one of the key issues of future research, and will contribute to obtain a deeper understanding of the Kuroshio and KE dynamics and predictability.

6. Conclusions

In this paper we have reviewed observational and modelling studies of very complex oceanographic phenomena associated with the low-frequency (interannual/decadal) variability of the bimodal character of the Kuroshio and of its extension jet (a WBC system of particular relevance for climate and ecology), and have analysed different mathematical approaches recently used to investigate their predictability. A wide variety of model
results (ranging from idealized quasi-geostrophic models to sophisticated eddy-resolving OGCMs) provide clear evidence that the bimodality of both the Kuroshio south of Japan and the KE have a predominant intrinsic origin, that is, they owe their existence mainly to physical mechanisms internal to the ocean system rather than to a direct effect of time-varying atmospheric forcing. In the understanding of these phenomena the use of the concepts and methods of nonlinear dynamical systems theory turn out to be crucial.

Variability of this kind is typically chaotic, hence its predictability is a delicate problem that must be tackled through specific mathematical approaches. We have reviewed applications of the CNOP method for the evaluation of the OPRs and OGIEs of the Kuroshio south of Japan. Applications of the Eulerian and Lagrangian Lyapunov exponents and of sequential importance sampling data assimilation methods are reviewed for the KE. In several cases, areas where targeted observations can improve the forecast have been identified.

Finally, the role played by surface wind stress changes in the central and eastern North Pacific Ocean in triggering the KE relaxation oscillation has been considered. The two KE bimodal cycles (starting roughly from 1995 and 2006, respectively) documented by altimeter data are found to be in synchrony with the main decadal time-scale modes of atmospheric variability. To understand the functioning of this mechanism the concepts of excitable system and of coherence resonance must be invoked. Recent studies provide a coherent picture of the complex interplay between intrinsic oceanic mechanisms and the effect of wind changes, and so reconcile two views that were previously considered to be competing. The recently proposed ocean-atmosphere feedbacks in WBC areas and the systematic identification of the intrinsic oceanic low-frequency variability on a global scale open new perspectives for a deeper understanding of these fascinating oceanographic phenomena.

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