Conditional nonlinear optimal perturbation of a T21L3 quasi-geostrophic model

Zhina Jiang, a,b Mu Mu, b* and Donghai Wang a

a State Key Laboratory of Severe Weather (LaSW), Chinese Academy of Meteorological Sciences, Beijing, China
b State Key Laboratory of Numerical Modelling for Atmospheric Sciences and Geophysical Fluid Dynamics (LASG), Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing, China

ABSTRACT: Conditional nonlinear optimal perturbation (CNOP) is defined as an initial perturbation that makes the cost function attain the maximum at a prescribed forecast time with physical constraint conditions, which is a natural generalization of the linear singular vector (LSV) into the nonlinear regime. In this paper, CNOPs of a T21L3 quasi-geostrophic (QG) spectral model in three kinds of norms are obtained numerically by solving nonlinear optimization problems, and further compared with their linear counterparts, namely LSVs.

Results reveal that CNOPs do depend, as LSVs do, on the norm chosen. The stream-function-squared norm yields small-scale disturbances; the results obtained by total-energy norm are characterized by intermediate-scale disturbances; and in the case of the enstrophy norm, CNOPs are typified by large-scale disturbances with large zonal flow contribution. If the linear approximation is valid, CNOP shows great resemblance to LSV, both of which are much localized in cases of the stream-function-squared and total-energy norms. However, with the increasing of the optimization time interval and/or the magnitude of the initial constraint condition, CNOP has less localized structures than the corresponding LSV in these two norms. What is more, the wave train structures of CNOP may even be found in the whole zonal direction in the Northern Hemisphere. The evolutions of perturbations, represented by CNOPs and LSVs in the total-energy norm, are also investigated in detail. The similarities and dissimilarities between CNOPs and LSVs are revealed not only from the growth rate but also from the similarity index of spatial patterns.

The numerical results imply that the method of CNOP may be a more appropriate tool for the study of stability and sensitivity problems when nonlinearity is of importance. Furthermore, the proper norm should be chosen for different physical problems. Copyright © 2008 Royal Meteorological Society

KEY WORDS conditional nonlinear optimal perturbation; linear singular vector; nonlinearity

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1. Introduction

A number of researches have investigated linear perturbations to atmospheric and oceanic flows that are determined from the constraint that they produce the maximum possible growth in a given norm over a finite interval of time (Farrell and Moore, 1992; Buizza et al., 1993; Buizza and Palmer, 1995; Ehrendorfer, 2000). This fastest growing perturbation, which is also called linear singular vector (LSV), was first examined by Lorenz (1965) in the meteorological research field. Since then, the LSV approach has been widely used in the study of predictability, ensemble prediction and adaptive observations (Molteni and Palmer, 1993; Molteni et al., 1996; Ehrendorfer and Tribbia, 1997; Palmer et al., 1998; Frederiksen, 2000; Buizza et al., 2005).

Note that LSV is established on the fact that the initial perturbation is sufficiently small and the time period is not too long, so that the evolution of the perturbation can be governed by the tangent linear model (TLM). However, there is no consensus about the forecast period for which the linearity assumption is valid (Lacarra and Talagrand, 1988; Errico et al., 1993; Mu et al., 2000). Keeping in mind that the motions of the atmosphere and the oceans are both dominated by complicated nonlinear systems and the linear assumption may have some limitations, Barkmeijer (1996) tried a modification technique to the linearly fastest-growing perturbation and attempted to construct fast-growing perturbations for the nonlinear regime. Nevertheless, he also recognized that his technique may not necessarily result in the nonlinearly fastest-growing perturbation. A similar procedure was described in Oortwijn and Barkmeijer (1995) and Oortwijn (1998). Then, Mu and Duan (2003) proposed a new method called conditional nonlinear optimal perturbation (CNOP), which can produce the maximum nonlinear evolution at a given forecast time with respect to a predefined norm. Duan et al. (2004), Mu et al. (2004), Duan and Mu (2006) and Duan et al. (2008) applied this method to study the predictability of El Niño–Southern Oscillation (ENSO) and sensitivity and stability of the ocean’s thermohaline circulation for simple models of
ordinary differential equations. Recently, CNOPs have been numerically obtained for models of partial differential equations with high dimensions. In Mu and Zhang (2006), CNOPs and LSVs in the total-energy norm were compared by using a two-dimensional barotropic quasi-geostrophic model from a methodological point of view. Mu et al. (2007a) obtained CNOPs in the Zebiak–Cane model. It was found that CNOP-type error tends to have a significant season-dependent evolution, and produces most considerable negative effects on the forecast results. Therefore, CNOPs are closely related to the spring predictability barrier (SPB).

Despite this flurry of recent activity, there is no systematic study on the sensitivity of CNOP to the choice of norms. Considering that LSV depends quite sensitively on the choice of norm, with different norms characterized by different-scale disturbances (Palmer et al., 1998; Frederiksen, 2000), we speculate that CNOP, as a natural extension of LSV into the nonlinear regime, may also have this property. So, the objective of the present paper is to examine the dependence of CNOP on the norm in a three-level global quasi-geostrophic model developed by Marshall and Molteni (1993), in which emphasis is placed on the effect of the nonlinear process on the structures and evolutions of CNOP compared with LSV. Here, we consider three norms, namely the total-energy, stream-function-squared and enstrophy norms. The corresponding nonlinear optimization problem with initial constraint condition is formulated, and the optimization algorithm employed is the spectral projected gradient (SPG2) (Birgin et al., 2000), which calculates the least value of a function of several variables subject to box or ball constraints. The great strength of the SPG2 method is its ability to solve problems with higher dimensions. Besides, this technique needs only to calculate the gradient of the objective function with respect to the initial perturbations, which is different from the sequential quadratic programming (SQP) method used in Mu and Zhang (2006).

The plan of this paper is as follows. In section 2, we introduce the T21L3 model and present the CNOP method. The three norms used are also illustrated here. The structures of CNOPs and LSVs in different norms and their nonlinear evolutions are examined in section 3. Discussion and conclusions drawn follow in section 4.

2. The model and method

2.1. The model

In our study of the predictability of atmospheric motion, we use a three-level global quasi-geostrophic (QG) spectral model, describing the evolution of the potential vorticity at 200, 500 and 800 hPa (Marshall and Molteni, 1993). It is triangularly truncated at wave number 21 (T21) including orography. The global model is driven by empirical forcing functions, which have been tuned to describe a perpetual winter situation in the Northern Hemisphere. The tangent linear version of the model and its adjoint are available. Till now, the model has been used for a large number of studies of predictability and data assimilation (e.g. Molteni and Palmer, 1993; Houtekamer and Derome, 1995; Vannitsem and Nicolis, 1997; Houtekamer and Mitchell, 1998; Ehrendorfer, 2000).

2.2. Theoretical background

In this subsection, we assume that for fixed $T > 0$ and initial potential vorticity $Q|_{T=0} = Q_0$, the propagator $N$ is well-defined; $Q(T) = N(T)Q_0$ is the solution of the nonlinear model at time $T$. Perturbations $q_0$ to the initial condition $Q_0$ result in deviations from the original trajectory, so that the system follows a new trajectory $ar{Q}(T) = N(T)Q_0 + q_0$. The nonlinear evolution of $q_0$ is defined as $q(T) = N(T)q_0 - N(T)Q_0$. We also assume that for sufficiently small perturbation, the perturbation $q(T)$ can be determined by integrating the tangent linear model for time $T$; $q(T) \approx M(T)q_0$, where $M$ represents the tangent linear propagator.

The following three kinds of norms are employed to compute the perturbation growth. The enstrophy norm is defined as

$$||q||_{E}^2 = <q, q> = \int qVq dV.$$  

The stream-function-squared norm is

$$||q||_{S}^2 = [q, q]_S = \int E^{-1}qE^{-1}q dV,$$

and total-energy norm is

$$||q||_{E}^2 = (q, q)_E = <q, -E^{-1}q> = -\int qV^{-1}q dV,$$

which are all integrated over the whole atmosphere $V$, where $q$ is the potential vorticity perturbation. Define $q = E\phi$ with the stream function perturbation $\phi$, where $E$ is the linear operator that transforms stream function into potential vorticity, and $E^{-1}$ is the inverse operator, which transforms potential vorticity into stream function.

2.3. The CNOP method

In the following, we take the total-energy norm as an example to introduce the procedure of acquiring CNOP. CNOP is the initial perturbation $q_{0\sigma}$ that makes the objective function $J(q_0)$ acquire the maximum value under the initial constraint condition $||q_0||_{E} \leq \sigma$:

$$J(q_{0\sigma}^*) = \max_{||q||_{E} \leq \sigma} J(q_0),$$

where

$$J(q_0) = ||N(T)Q_0 + q_0 - N(T)Q_0||_{E}.$$  

$Q_0$ and $q_0$ are the initial basic state and perturbation respectively, and $\sigma$ is a presumed positive constant.
representing the magnitude of the initial uncertainty, whose dimensional value is in units of energy per unit mass. Note that the norms used in the objective function and the initial constraint condition are the same.

To capture the maximum of \( J(q_0) \) with the constraint \( ||q_0||_E \leq \sigma \), we calculate the minimum of a new objective function with the same constraint \( ||q_0||_E \leq \sigma \). The new objective function is defined as

\[
J_1(q_0) = -||J(q_0)||_E^2 = -||N_T(Q_0 + q_0) - N_T(Q_0)||_E^2.
\]

(6)

The first variation of \( J_1(q_0) \) is

\[
\delta J_1(q_0) = -\left[2M^*E(Q_0 + q_0)\right]^{T} \times \left\{N_T(Q_0 + q_0) - N_T(Q_0), \delta q_0\right\}_E.
\]

(7)

In the numerical models used for this study, the adjoint \( M^* \) of the tangent version of the model equations has been defined with respect to the enstrophy inner product. According to Buizza et al. (1993), the adjoint operator \( M^*E \) with respect to the above total-energy norm can be deduced from \( M^* \). Hence, \( M^*E = EM^*E^{-1} \).

Then,

\[
\delta J_1(q_0) = -\left[2EM^*(Q_0 + q_0)\right]^{T} \times \left\{N_T(Q_0 + q_0) - N_T(Q_0), \delta q_0\right\}_E
\]

\[
= (\nabla J_1, \delta q_0)_E,
\]

(8)

where \( \nabla J_1 \) is the gradient of the new objective function \( J_1 \) with respect to initial perturbation \( q_0 \). The above deductions have provided the conditions to use the SPG2 method, and then the nonlinear optimization problem can be solved numerically. According to the definition of CNOP, CNOP is the global maximum of the cost function. However, there exists the possibility that the cost function attains its local maximum in a small neighbourhood of a point in the phase space. Such an initial state is called local CNOP, which, in some cases, possesses clear physical meaning. For example, Duan et al. (2004) revealed that global CNOP (local CNOP) acquired on the climatological background state is most likely to evolve into El Niño (La Niña) event and acts as the optimal precursors for El Niño (La Niña).

3. Numerical results

In this section, we examine CNOPs and LSVs and their time evolutions for three basic states, which are characterized by different large-scale circulations. The dependence of CNOPs on the norm is studied with different magnitudes of initial constraint conditions and time periods, so as to explore the effect of nonlinearity on the structures and evolutions of CNOPs. The results are further compared with those of LSVs.

Numerical experiments show that CNOPs including local CNOPs are all on the boundaries of the initial constraint. Hence, the norms of CNOP and local CNOP are equal to \( \sigma \). To compare with the corresponding CNOP, the first LSV is also generated using the SPG2 method with the smallest constraint condition, rather than an iterative power method adopted by Farrell and Moore (1992). Here, the objective function for LSV is a modified version of CNOP, which is obtained by replacing the nonlinear evolution of the initial perturbation by integrating the tangent linear model. Due to the linear characteristics of LSV, we can create another LSV by multiplying the original LSV by a constant, so as to make its norm also equal to \( \sigma \).

The similarity index \( S \), according to Buizza (1994) and Kim et al. (2004), is defined to quantify the degree of similarity between two patterns:

\[
S = \frac{|e_1, e_2|}{||e_1||_{L^2} ||e_2||_{L^2}},
\]

(9)

where \( e_1 \) and \( e_2 \) represent the stream function fields of pattern one and pattern two, respectively. From the above formula, it is seen that \( S \) ranges from \(-1 \) to \(1 \).

3.1. Experiment 1

The first calculation is made from time-evolving trajectories beginning on the ECMWF (European Centre for Medium-Range Weather Forecasts) analysis of 00 UTC 9 January 1993, which is characterized by a blocking high over the northeast Pacific.

3.1.1. The case of enstrophy norm

Figure 1 presents the geopotential perturbation fields of initial and evolved LSV and CNOPs in the enstrophy norm at 500 hPa with \( \sigma = 1.0 \times 10^{-7} \, (s^{-1}) \) for 2 days. The geopotential perturbation fields are obtained from the stream function by solving the linear balance equation. In this case, local CNOP is also found. One sees that in the initial states, all the geopotential perturbation fields exhibit nearly circumpolar zonal flow structures. As the disturbances amplify, they develop into wave train structures across the Northern Hemisphere midlatitudes, which increasingly become concentrated over the Pacific Ocean and into the European sector. What is more, global CNOP has quite similar initial and final structures to LSV, which means that the linear approximation is valid under this condition. Besides, the spatial pattern of local CNOP is quite similar to that of the negative LSV. That is to say, the global and local CNOPs exhibit symmetric structures in this case, just like LSV and the negative LSV.

For the same initial constraint condition \( \sigma = 1.0 \times 10^{-7} \, (s^{-1}) \), we explore the effect of the optimization time interval on the spatial patterns of LSV and CNOP. Figure 2 shows the geopotential perturbation fields of LSV and CNOPs in the enstrophy norm at 500 hPa for 7 days. We find that global CNOP exhibits a positive anomaly centre away from the polar region, and its amplitude is smaller than that of LSV. Whereas local CNOP shows a negative anomaly centre at almost the...
1.0 × 10^{-7} (s^{-1}) for Experiment 1. (a) LSV; (b) linear evolution of (a); (c) global CNOP; (d) nonlinear evolution of (c); (e) local CNOP; (f) nonlinear evolution of (e).

Figure 1. The geopotential perturbation fields of the initial and evolved LSV and CNOPs in the enstrophy norm for 2 days with $\sigma = 1.0 \times 10^{-7} (s^{-1})$ for Experiment 1. (a) LSV; (b) linear evolution of (a); (c) global CNOP; (d) nonlinear evolution of (c); (e) local CNOP; (f) nonlinear evolution of (e).

same region as global CNOP, however, its amplitude is larger than that of the negative LSV. The similarity index between global and local CNOPs is calculated to be $-0.928$. It seems that the effect of nonlinearity on global and local CNOPs makes them exhibit asymmetric structures compared with LSV and the negative LSV.

3.1.2. The case of stream-function-squared norm

If the optimization time interval is 2 days, and the initial constraint condition $\sigma = 4.0 \times 10^4 (m^2 s^{-1})$, one can see that the global CNOP in the stream-function-squared norm has small-scale baroclinic wave train structures, localized in the European sector, which shows great resemblance to the corresponding LSV. Besides, the spatial pattern of local CNOP is also very similar to that of the negative LSV (not shown).

Note that the magnitude of initial constraint has no effect on the spatial pattern of LSV, but influences that of CNOP. Then, we increase the magnitude of the initial constraint condition with $\sigma = 6.0 \times 10^5 (m^2 s^{-1})$ to see
what will happen. It is found that CNOPs and LSV show great differences. The similarity index between global CNOP (local CNOP) and LSV is 0.742 (0.724). Figure 3 shows the geopotential perturbation fields of LSV and CNOPs in the stream-function norm at 500 hPa for 2 days. The global and local CNOPs have less localized structures than the corresponding LSV. Moreover, the main wave train structures of the global CNOP shift to the North American and North Atlantic areas. As for the local CNOP, main wave train structures can be found not only in the European sector but also in the Asian area.

3.1.3. The case of total-energy norm

Also, if the optimization time interval is 2 days and the initial constraint condition \( \sigma = 0.01 \text{ (J kg}^{-1}\) ), the numerical results show that the linear approximation is still valid. Figure 4 shows the LSV and CNOPs at 500 hPa with \( \sigma = 3.0 \text{ (J kg}^{-1}\) for 2 days in the total-energy norm. In this case, the linear approximation is not valid; the similarity index between global CNOP (local CNOP) and LSV is only 0.536 (0.610). It is also found that LSV is much localized in the North America and North Atlantic areas. Comparing with LSV in the stream-function norm shown in Figure 3, apart from the different location, one can see that the initial CNOPs and LSV in the total-energy norm have considerably larger scale. In like manner, CNOPs in the total-energy norm have less localized structures than the corresponding LSV in the same norm, which extend the wave trains even into the whole zonal direction.

The differences between CNOPs and LSVs can also be seen from their evolutions. Table I summarizes the results of the global CNOPs and LSVs in the total-energy norm with \( \sigma = 0.01, 1.0, 3.0, 5.0 \text{ (J kg}^{-1}\) for 2 days. For brevity, the local CNOP and the negative LSV are not listed here. \(|q_{LL}|_E\) and \(|q_{LN}|_E\) represent the energy norms of the linear and nonlinear evolutions of LSV, respectively. \(|q_{TL}|_E\) and \(|q_{TN}|_E\) are the energy norms of the linear and nonlinear evolutions of the global CNOP, respectively. It can be seen that the evolutions of global CNOP are almost the same as those of LSV when the constraint \( \sigma \) is very small. With increasing \( \sigma \), the nonlinear evolution of the global CNOP increases. The larger \( \sigma \) is, the larger the difference between global CNOP and LSV becomes. In addition, the nonlinear evolution of the global CNOP is larger than that of LSV, and its linear evolution is smaller than that of LSV, which is consistent with their respective definitions. Furthermore, we also find that the nonlinear evolution of global CNOP is smaller than the linear evolution of LSV, which shows that the effect of the nonlinearity on the evolving basic state may be to reduce perturbation growth. The detailed process of the nonlinear interaction is beyond the scope of this paper.
Figure 3. The geopotential perturbation fields of LSV and CNOP in the stream-function-squared norm for 2 days with $\sigma = 6.0 \times 10^5$ $(m^2 s^{-1})$.
(a) LSV; (b) global CNOP; (c) local CNOP.

Figure 4. The geopotential perturbation fields of LSV and CNOPs in the total energy norm for 2 days with $\sigma = 3.0$ (J kg$^{-1}$). (a) LSV; (b) global CNOP; (c) local CNOP.
Table I. Linear and nonlinear evolutions of LSVs and global CNOPs in the total-energy norm for 2 days under different initial constraint conditions for Experiment 1.

<table>
<thead>
<tr>
<th>$\sigma$ (J kg$^{-1}$)</th>
<th>0.01</th>
<th>1.0</th>
<th>3.0</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td></td>
<td>q_{LL}</td>
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<td>q_{TL}</td>
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<td>_E$</td>
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<tr>
<td>$</td>
<td></td>
<td>q_{TN}</td>
<td></td>
<td>_E$</td>
</tr>
</tbody>
</table>

3.2. Experiment 2

The second basic state is also characterized by a blocking high, but existing over the European–Asian areas, which begins on the ECMWF analysis of 00 UTC 2 December 1993. CNOPs and LSVs in these three norms are also numerically obtained.

As shown above, the norm dependence of CNOP is also revealed from the aspects of the spatial pattern of initial perturbations. If the optimization time interval...
or the magnitude of the initial constraint condition is not so small, TLM is not a good approximation for describing the nonlinear evolution of initial perturbations. The method of CNOP provides an efficient means for capturing the nonlinear characteristics of the system. Here, for brevity, only CNOPs in the total-energy norm are further illustrated.

Also, CNOP and LSV in the total-energy norm show great resemblance with $\sigma = 1(\text{J kg}^{-1})$ for 2 days, both of which are localized in the Asian area.

Figure 5 shows the LSV and global and local CNOPs in the total-energy norm with $\sigma = 1(\text{J kg}^{-1})$ for 5 days at 200 hPa, 500 hPa and 800 hPa respectively. We find that with the increasing of the optimization time interval, LSV has the appearance of extended wave train structures, but still exhibits localization. However, the global and local CNOPs almost lie in the whole zonal direction in the Northern Hemisphere. Baroclinically amplifying patterns with a westward tilt with height appear in all the initial perturbation fields. The similarity index between global CNOP (local CNOP) and LSV is only 0.322 (−0.526).

The evolved disturbances at optimization time, which are shown in Figure 6, all have equivalent barotropic structures, which increasingly become concentrated over the North Pacific Ocean and into the American sector.

Figure 5. (Continued).

Table II gives the linear and nonlinear evolutions of the LSVs and global CNOPs in the total-energy norm with $\sigma = 1.0(\text{J kg}^{-1})$ under different optimization time periods. It is clear that with the optimization time periods increasing, the nonlinear evolution of global CNOP increases. The characteristics of CNOPs are similar to those summarized in Table I.

### 3.3. Experiment 3

For the last case, a zonal flow in the Northern Hemisphere is chosen, which begins with the ECMWF analysis of 00 UTC 10 December 1993. Following this control basic state, during the integration time interval no blocking high occurs. Similarly, the cases of three different norms are studied.

Firstly, the LSV and global and local CNOPs in the enstrophy norm with $\sigma = 1.0 \times 10^{-7} (\text{s}^{-1})$ for 7 days are obtained. We find that the amplitude of global (local) CNOP is smaller (larger) than that of LSV. The similarity index between global and local CNOPs is up to −0.996. The global and local CNOPs show asymmetric structures compared with LSV and the negative LSV.

In the case of the stream-function-squared norm, LSV and global and local CNOPs with $\sigma = 4.0 \times 10^{5} (\text{m}^{2}\text{s}^{-1})$ for 5 days at 800 hPa are presented in Figure 7. One sees that CNOPs in the stream-function-squared norm are typically characterized by small-scale wave train structures, which are less localized than the corresponding LSV. The similarity index between global CNOP (local CNOP) and LSV is 0.504 (0.248).

The LSVs and global and local CNOPs in the total energy norm with $\sigma = 1.0 (\text{J kg}^{-1})$ for 7 days are also obtained. The similarity index between global CNOP (local CNOP) and LSV is only 0.220 (0.483).

Table II. Linear and nonlinear evolutions of LSVs and global CNOPs in the total-energy norm with $\sigma = 1.0 (\text{J kg}^{-1})$ under different optimization time intervals for Experiment 2.

<table>
<thead>
<tr>
<th>$T$ (days)</th>
<th>2</th>
<th>3</th>
<th>5</th>
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<tr>
<td>$</td>
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<td>q_{LL}</td>
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<td>q_{TN}</td>
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It is worthwhile to point out that in the above three cases, the nonlinearity plays an important role, which makes the two kinds of initial perturbations show great differences. Furthermore, as in the above two experiments, there is typically an upscale evolution from subsynoptic to synoptic scale between CNOP at initial time and optimization time using the total-energy and stream-function-squared norms. By contrast, initial CNOP produced in the enstrophy norm tends to be dominated by planetary-scale perturbations. During evolution, enstrophy generation is maximized by the downscale cascade to synoptic scales between initial and optimization time.

4. Concluding remarks

LSV is the fastest growing perturbation at a prescribed forecast time in the linear regime; its structure may not be optimal when interactions between the perturbation and the basic state become large. In this paper, CNOPs of a T21L3 QG model are obtained by solving the corresponding nonlinear optimization problem numerically. We have
explored the dependence of CNOP on the choice of norm and focused on the role of nonlinearity in determining the structures and evolutions of CNOP compared with LSV. The main conclusions can be summarized as follows.

CNOPs, as well as LSVs, depend sensitively on the choice of norm with the stream-function-squared norm characterized by small-scale disturbances, the total-energy norm giving intermediate-scale disturbances, and the enstrophy norm typified by large-scale disturbances with large zonal flow contribution. In cases of the stream-function-squared and total-energy norms, LSVs possess the property of localization. Whether CNOPs in these two norms also have this property depends on the optimization time interval and the magnitude of the initial constraint condition. When the nonlinearity is of importance, CNOPs in these two norms show great differences from LSVs in spatial patterns, which have less localized structures than the corresponding LSVs, and wave train structures can even be found in the whole zonal direction in the Northern Hemisphere. Meanwhile, the effect of nonlinearity on global and local CNOPs in different norms makes them exhibit asymmetric structures compared with LSV and the negative LSV.
Figure 7. The geopotential perturbation fields of LSV and CNOPs in the stream-function-squared norm for 7 days with $\sigma = 4.0 \times 10^5 \text{m}^2 \text{s}^{-1}$ at 800 hPa for Experiment 3. (a) LSV; (b) global CNOP; (c) local CNOP.

In the idealized situation where the atmospheric circulation is measured at the nodes of a uniform grid, measurement errors are dominated by their subgrid-scale components. The total-energy and stream-function-squared norms are therefore more relevant to describe error growth, which is a subtle reason for preferring these norms in the study of predictability of weather (Buizza and Palmer, 1995; Ehrendorfer, 2000). Based on different sets of calculations, Palmer et al. (1998) argued that for predictability studies, a first-order approximation to the appropriate metric can be based on perturbation energy. Relating CNOP sensitivity to the choice of norm, a suitable inner product should be chosen based on different physical problems.

As demonstrated by Tables I and II, in the total-energy norm, when the linear approximation is valid, the evolutions of CNOP show great resemblance to those of LSV. However, with the increasing of the optimization time interval or the magnitude of initial constraint condition, the evolutions of CNOP show differences from those of LSV. The nonlinear evolution of CNOP is smaller than the linear evolution of LSV, which shows that the effect of the nonlinearity on the evolving basic state may be to reduce perturbation growth; this appears to be the consequence of saturation arising from the dynamical conservation properties of the equations of motion. Besides, the nonlinear evolution of CNOP is larger than that of LSV, which is consistent with the definition of CNOP. We speculate that the method of CNOP could be useful in composing an ensemble perturbation that produces a significantly larger spread than LSV in the medium range. It also shows that if the ensemble prediction system adopts an alternative set of initial perturbations generated by our method, of which the objective function may be reconstructed according to the physical problems, a possible regime transition can be determined more directly (Molteni and Palmer, 1993; Oortwijn and Barkmeijer, 1995). Certainly, how to combine the CNOP method with LSV or other methods to make the ensemble perturbations is a valid question to be studied. Besides, different magnitudes of initial constraint and optimization time intervals may be other ways of generating more ensemble perturbations.

Considering the potential use of LSVs in defining a strategy for utilizing technology for making adaptive or targeted observations of the atmosphere (Palmer et al., 1998), the CNOP method is encouraged to be applied in this field (Mu et al., 2007b). However, how to specify the targeting area when CNOPs are available and whether there is another better choice of norm in the definition of CNOP, etc., all needed to be studied further.
Besides, it is worthwhile to point out that global and local CNOPs of the T21L3 QG model are all located at the boundaries of the domain defined by the constraint conditions in the phase space, which confirms the conjecture by Mu and Zhang (2006). The nonlinear optimization algorithm SPG2 used in this paper proved to be a successful one in higher dimensional problems. Recently, the adjoint versions of many atmospheric and oceanic models have been developed for data assimilation, which provides conditions for the CNOP method to be widely employed.

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References


