Flexible metamaterial narrow-band-pass filter based on magnetic resonance coupling between ultra-thin bilayer frequency selective surfaces
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Abstract. A novel flexible metamaterial narrow-band-pass filter is designed and proved to be reliable by both numerical simulations and experimental measurements. The unit cell of the designed structure consists of circle ring resonators on the top of a thin dielectric layer backed by a metallic mesh. The investigations on the distribution of the surface current and magnetic field as well as the analysis of equivalent circuit model reveal that the magnetic resonance response between layers induced by the reverse surface current contributes to the high quality factor band-pass property. Importantly, it is a flexible design with tunable resonance frequency by just changing the radius of the circle rings and can also be easily extended to have the multi-band-pass property. Moreover, this simplified structure with low duty cycle and ultra-thin thickness is also a symmetric design which is insensitive to the polarization and incident angles. Therefore, such a metamaterial narrow-band-pass filter is of great importance in the practical applications such as filtering and radar stealth, and especially for the conformal structure applications in the infrared and optical window area.

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1. Introduction

In the past decade, the artificial electromagnetic (EM) materials known as metamaterials (MMs), which are designed with periodic sub-wavelength structures, have attracted more and more attentions due to their unprecedented properties compared to the natural materials. Providing the ability of tailoring the effective permittivity and permeability independently, MMs have varieties of immediate and potential applications, such as negative refractive index materials [1-5], perfect lens [6-8], invisible cloaking [9-14] and perfect absorbers [15-20] with a wide range of frequencies from microwave towards visible. Frequency selective surface (FSS), a two dimensional structure composed of a periodic array of metallic units, has been the subject of intensive investigation for its important applications as spatial and optical filters with band-stop or band-pass capability for nearly five decades [21-25]. Combining the advantages of the above two special structures, MMs containing FSSs have such an excellent resonance frequency stability and tunability performance that they have played a key role in the EM filtering area.

In modern wireless communication systems, high-performance narrow-band RF/microwave band-pass filters (BPFs) with high selectivity are required to shield the out-of-band noise and improve the signal to noise ratio (SNR). It is also extremely significant to reduce the size and weight in order to integrate them with other components as a compact system. A planar MM BPF has been a good candidate for this purpose and lots of relative works have been carried out, such as substrate integrated waveguide (SIW) cavity FSS [26], complementary composite MM [27] and miniaturized-element FSS consisting of capacitive patches and inductive wire grid acting as a parallel coupled LC circuit [28-30]. Recently, Bayatpur and Sarabandi have proposed a novel FSS with a two-dimensional period array of metallic loop and grid, which shows a good band-pass property [31]. However, there are still some deficiencies for its practical application. On account of the same period of the two layers, the flexibility of the whole structure is strongly limited for the single parameter tuning of the pass-band and a higher quality-factor (Q). Moreover, for some applications such as EM shielding window, the need for high transparency in the optical/IR range still can’t be satisfied by this structure due to its large wire width and relatively high duty cycle (i.e. the ratio of the area covered with metal wires to the whole area of the surface). Considering these issues, a BPF with high flexibility and low duty cycle for the optical window applications is highly desired.

In this paper, a MM BPF with more simplified and flexible design is presented. The structure consists of bilayer FSSs with circle ring resonators at the front and a metallic mesh layer at the back, which are separated by a thin dielectric spacer. By meeting the impedance matching of the interface between the MM and the free space and simultaneously minimizing the transmission loss, the as-proposed MM BPF structure have a flexibly tunable narrow-band-pass filtering property with a broad-band shielding out of the pass-band. Moreover, this structure can also be easily extended to have a multi-band-pass character using the concentric rings on the front layer. Upon the more systematic investigation and analysis of the three-dimensional (3D) electric-magnetic field distribution, a magnetic resonance coupling mechanism between the FSS layers is proposed and the related equivalent circuit model is further...
employed to explain the characteristics of the parametric variations, which correspond with the simulation and calculation results. Importantly, the experimental measurement results are consistent well with the numerical simulations. In addition, our design is polarization independent and insensitive to the incident angle due to the geometrical symmetry and has a very low duty cycle for its extremely simple geometric configurations and ultra-thin character, which is of vitally practical significance for the infrared and optical window.

2. Sample design, simulation and measurement

A simple and ultra-flexible design is introduced in our MM BPF structure, as shown in Figure 1, which consists of three layers: the periodic arrays of circle ring resonators at the front layer, a metallic mesh at the back layer, and a thin dielectric spacer sandwiched between the former two layers. The corresponding parameters of this structure are also shown in Figure 1. The circle period (P) is 8 mm (five times of mesh grid) and the dielectric thickness (h) is 0.2 mm. Besides, the metal line widths (w) of both the front and back layers are 0.2 mm. The basic unit of the whole structure includes one circle ring with the initial radius (R) 2.8 mm and 5×5 metal wires. Therefore, this structure is not only simplified with a very low duty cycle, but also has a central symmetrical feature along the z axis.

The numerical simulations were performed with the 3D full-wave EM simulation software (CST Microwave Studio), based on the finite integration method. To calculate the S-parameters, Floquet periodic boundaries and added open space were set in the x-y plane and z direction as the unit cell conditions, respectively. The TE and TM polarizations of the incident plane-wave were chosen in such a way that the electric field was oriented along y and x directions, respectively. All the metals were made by copper with a conductivity of 5.8×10^7 S m⁻¹, and to minimize to absorption loss of the substrate, Rogers 4003C was chosen as the dielectric layer material from the CST material library with a dielectric constant of 3.55 and a quite low loss tangent of 0.0027. To explore the band-pass filtering behavior, we focus on the transmission coefficient (S₂₁) of this MM BPF structure.

![Figure 1](image.png)
As for the experimental measurement process, a 25×25 unit-cells prototype was manufactured by the conventional printed-circuit-board (PCB) process with copper patterns (18 μm thick) on both sides of a commercial used Rogers 4003C dielectric substrate, as shown in Figure 2(a). The transmission magnitude was measured in free space using a network analyzer setup (Agilent Technologies PNA-X N5230A) and a pair of horn antennas (the scanning frequency range is 1-18 GHz) of which the receiving antenna is packaged in the microwave anechoic chamber, as illustrated in Figure 2(b). To avoid the near-field effects, the height of the system was 1.5 m and the sample was set between the two antennas with a horizontal distance of 0.5 m away from the transmitting antenna and 0.3 m away from the receiving antenna. It should be noted that the data was calibrated before each test.

3. Results and Discussion

Figure 3(a) shows the simulated and measured $S_{21}$ parameters of the MM BPF for TE polarization at the normal incidence. According to the simulated result, our proposed structure shows a distinct narrow-band-pass property in the radar frequency band from 1 to 18 GHz, with the transmittance peak at 9.52 GHz. The experimentally measured result of the sample is consistent well with the simulated value, though the transmission peak slightly moves to 9.67 GHz and the value of the maximum transmittance is about 2 dB lower than that obtained by the simulation. It is believed that the main reason for the transmission attenuation is the larger substrate loss in the experiment than that in the simulation due to the possible errors of the material composition or processing during its preparation, which is unavoidable. While the frequency discrepancy is mainly ascribed to the fabrication tolerance, as well as the dielectric board material whose actual dielectric constant is slightly different from the value used in the simulations. Furthermore, the discrepancies between the simulation and measurement can also be attributed to the fact that in the simulation using CST Microwave Studio, periodic boundary conditions are set, indicating the physical size of the proposed absorbers is infinite, while the dimension of the fabricated sample in the experiments is finite, resulting in the edge diffraction. Considering these factors, the narrow-band-pass property of our proposed structure has been proved by both simulation and experiment results. Further, to quantitatively describe the this filtering property, Q-factor, defined as the ratio of the central frequency to the full width at half

\[ Q = \frac{f_0}{\Delta f} \]

where $f_0$ is the central frequency and $\Delta f$ is the full width at half maximum (FWHM) of the peak transmittance.
maximum (FWHM) bandwidth of a resonance from the simulated normalized transmission coefficient ($S_{21}$) as shown in Figure 3(b), has also been calculated with the value of 20.039, which is higher than the recently reported result of 17.1 [32].

![Graph showing simulated and measured transmission coefficients](image)

**Figure 3.** (a) Simulated and measured transmission coefficients ($S_{21}$) at the normal incidence and (b) the simulated normalized $S_{21}$ of the MM filter; (c) the real part (red line) and imaginary part (black line) of the effective impedance.

In order to gain a better insight into the underlying physics, the numerical simulation is also employed in the EM characteristics analysis of our MM BPF. Figure 4 illustrates the surface current and magnetic field distributions of the proposed unit cell (dimensions are as reported in Figure 2) induced by a uniform TE plane wave incident in the z direction. By comparing the surface current distribution at two special resonance points (transmission peak at 9.52 GHz and rejection peak at 10.58 GHz of the simulated $S_{21}$ curve in Figure 3), it is clear to see two different electromagnetic resonance modes, which lead to the inverse transmittance responses: band-pass and band-stop. On one hand, at the resonance point of 9.52 GHz, the enhancement of the surface current intensity on the top surface (circle rings) distributes “closely” to that on the bottom surface (mesh), as illustrated in Figures 4(a) and 4(c). The surface current vectors and magnetic magnitude distribution shown in Figures 4(e) and 4(g) indicate that the magnetic resonance is induced in the middle dielectric layer by the reverse surface current on the adjacent parts of the two conductive FSS layers. When both the electric field and magnetic field resonate at the same frequency, they will collaboratively cause the impedance matching at the interface between the MM and the free space, so that the reflectance will be zero [33]. Besides, as the absorptance of the multilayers is minimized to 0, the designed structure can correspondingly function as a band-pass filter. On the other hand, at the resonance point of 10.58 GHz, the distribution of surface current on the two separated metallic layers is totally different in the x-y plane. Especially, the surface current on the bottom layer, mainly induced on both sides in the y direction of the unit cell, is so far from that of the top layer (Figures 4(b) and 4(d)) that the reverse surface current interaction between the two layers is too weak to induce the magnetic resonance in the adjacent dielectric spacer (Figures 4(f) and 4(h)). Therefore, it can be regarded as an electrical negative MM structure which will lead to the band-stop character. To prove the above discussion,
the equivalent impedance factor derived by the methods of S-parameter retrieval [34] has also been calculated and shown in Figures 3(b) and 3(c). It is obviously that the real part of impedance is nearly 1 at 9.53 GHz, matching that of free space, while the image part is close to 0.

![Figure 4](image)

**Figure 4.** The surface current intensity distribution on the circle rings (a, b) and the bottom mesh (c, d) at 9.52 GHz (a, c) and 10.58 GHz (b, d); and the distribution of the surface current vectors on both metallic layers at 9.52 GHz (e) and 10.58 GHz (f), and the magnetic field intensity distribution in the dielectric layer at 9.52 GHz (g) and 10.58 GHz (h).

As illustrated above, the magnetic field is strongly induced in the dielectric spacer underneath the circle rings at the band-pass resonance point. Under this condition, the MM BPF structure can be approximately regarded as a series of metal wire pairs from the top and bottom layers. Hence, regardless of the resistance loss influence, the resonance condition of the magnetic response can be predicted by the simplified equivalent LC circuit model, as shown in Figure 5, where $L_m$ is the effective inductance of the metal wire pairs, $C_m$ accounts for the capacitance of the efficient conducting part of each circle ring on the top layer, and $C_g$ denotes the capacitance in the dielectric spacer between two adjacent FSS layers.

![Figure 5](image)

**Figure 5.** Schematic of the simplified equivalent LC circuit for the MM BPF.

According to the simplified equivalent circuit model illustrated in Figure 5, the total impedance can be expressed as [35].
\[ Z_{\text{tot}} = \frac{i\omega L_m}{1 - \omega^2 C_m L_m} - \frac{2i}{\omega C_g} + i\omega L_m \]  

(1)

The magnetic resonance that leads to the band-pass character occurs when \( Z_{\text{tot}} = 0 \), thus the magnetic resonance condition can be derived as,

\[ \omega_R = \left[ \left( C_g + C_m - \sqrt{C_g^2 + C_m^2} \right)/(L_mC_mC_g) \right]^{1/2} \]  

(2)

Obviously, the band-pass resonance frequency \( \omega_R \) is determined by the efficient inductance and capacitance \((L_m, C_m \text{ and } C_g)\). Since \( L_m \) is proportional to the effective length \( l_{\text{eff}} \) of the magnetic polaritons along the E-field direction at the resonant frequency, both \( C_g \) and \( C_m \) are proportional to \( l_{\text{eff}} \) similarly, thus the magnetic resonant frequency is related to \( l_{\text{eff}} \), which can be depicted as \( \omega_R \propto 1/l_{\text{eff}} \). According to the surface current distribution on the circle rings of the top layer, \( l_{\text{eff}} \) is proportional to the radius of the rings \( R \). Therefore, it is concluded that the resonance frequency is inversely proportional to \( R \).

**Figure 6.** (a) Simulated transmission coefficient (\( S_{21} \)) of the MM filter at the normal incidence for different radius of the circle rings on the top layer; (b) relationship between the band-pass resonance frequency and the radius of the circle ring on the top layer.

Moreover, the as-proposed MM BPF is also shown to have a flexible structure with the resonance frequency tunable in a long range by just changing the radius of circle rings on the top layer, as illustrated in Figure 6(a). It is seen that with the increase of the radius the transmission peak moves to lower frequency in the range from 16 GHz to 6.5 GHz, therefore an desired transmission frequency can be easily obtained. The relationship between \( \omega_R \) and \( R \), related to the data in Figure 6(a), has also been plotted in Figure 6(b). By the method of the least square fitting (LSF), the relational expression between the two parameters can be deduced as

\[ \omega_R = \frac{2\pi a}{R^b} \]  

(3)

which is consistent with the result of the LC equivalent circuit model with constant \( a \) of 26.73779.

Since the pass-band frequency is solely determined by the radius of the circle rings, to further embody the flexibility of our proposed MM BPF, the single circle ring
on the front layer in one unit is then extended to double, three or more concentric rings with different radius (other structural parameters remain unchanged), as a result of which, the multi-narrow-band-pass filtering property of the extended structures can be easily achieved. Taking the dual-ring (with the radius of the inner ring of 2.5 mm and the outer ring of 3.4 mm) and tri-ring (with the radius of the rings from inside to outside of 2.0 mm, 2.7 mm and 3.6 mm, respectively) structures as examples, the dual-band and tri-band MM BPFs have been demonstrated, as shown in Figures 7(a) and 7(b), respectively. The results from the experimental measurements and the numerical simulations have shown a good agreement despite of some minor discrepancies which are caused by the machining and test errors. Moreover, the pass-band position is proved to be adjusted independently by just changing the radius of the corresponding rings. Therefore, it is believed that the excellent flexibility and extensibility of our proposed MM BPF structure will make it be more practically significant in the potential applications.

Figure 7. Simulated and measured transmission coefficients ($S_{21}$) of the MM multi-BPF with the (a) dual-ring or (b) tri-ring structure on the top layer at the normal incidence. Inset: 3D view of the unit, respectively.

Now we have demonstrated that the resonance frequency can be tuned in a long range by just changing the radius of circle rings on the top layer. Thus, the major contribution of the front circle ring is its ability to adjust the bandpass resonance frequency flexibly without any other change. As for the contribution of the backed copper mesh, the copper mesh can work similarly as a metal-backed substrate in the metamaterial absorbers [36-38], which can prevent the EM wave from passing through and simultaneously enhance the EM field resonance. As for our proposed structure, the backed copper mesh has the advantage of a very low duty cycle which makes EM wave at the resonance frequency pass through the structure with a broad out-of-band shielding property.

The proposed MM BPF is a four-fold symmetry structure and has been found to be stable under different polarization angles at the normal incidence, as shown in Figure 8(a). It benefits from the fact that the structural features are identical under both the TE and TM plane wave modes and the arbitrary incident plane wave can be decomposed into the two modes, so it can effectively excite the corresponding surface current oscillation and magnetic resonance at the resonant frequencies, which will further interact with resonant electric filed. Moreover, the stability of narrow-band-
pass character under different incident angles $\theta$ for the TE configuration in Figure 8(b) and TM configuration in Figure 8(c) has been further numerically investigated. It can be seen that the transmission character is insensitive to the variation of the incident angle from $0^\circ$ to $60^\circ$ despite that the transmission peak moves slightly to the higher frequency and the pass-band width narrows down with the increase of the incident angle ((Figures 8(d) and 8(e)), which can be explained as follows. On one hand, as the incident angle increases, the effective length $l_{\text{eff}}$ of the magnetic polaritons along the E-field direction reduces slightly, and then leads to a slight increase of the resonance frequency $\omega_R$, which is similarly as discussed above. On the other hand, the efficient inductance of the magnetic polaritons also enlarges as the incident angle increases due to the same reason. Since the bandwidth $\Delta\omega_R$ is inversely proportional to $\sqrt{L_m}$ [39], it can thus be concluded that the larger the incidence angle is, the narrower the pass-bandwidth and the higher the Q-factor will be. Notably, the present simulation results have demonstrated that the as-proposed MM BPF has a polarization independent and wide incident angle narrow-band-pass filtering property for both TE and TM polarizations. In addition, if the MM BPF is inversely used, the band-pass filtering property remains both numerically and experimentally. Therefore, the proposed MM BPF is also significantly meaningful in the bidirectional applications.
Figure 8. Simulated transmission coefficient ($S_{21}$) of the MM filter at the normal incidence for different azimuthal angle $\phi$ (a) and at different incident angles $\theta$ for (b, d) TE and (c, e) TM polarization;

4. Conclusion

A flexible MM BPF has been proposed based on the compact bilayer FSSs and demonstrated to have excellent filtering properties by both the numerical simulations and experimental measurements. The magnetic coupling resonance principle has been discussed thoroughly by investigating on the distribution of the surface current and magnetic field as well as the analysis of the equivalent circuit model. Moreover, the transmission frequency is easily tunable in a long range of the RF band and the MM BPF can be easily extended to have the multi-band-pass property, as a result of the extremely flexible character of our proposed structure. In addition, it is not only an ultra-thin structure with its total thickness of only about 200 \textmu m (nearly 6/1000 of the resonance wavelength), but also has a very low duty cycle due to its simple structure only made up of loosely arranged thin metal lines. It is also demonstrated that this symmetric structure is insensitive to the polarization and incident angles. Based on these excellent properties, this MM BPF is of great importance in the practical applications such as electromagnetic shielding, filtering and radar stealth, and especially for the conformal structure application in the infrared and optical window area.

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