Discrete-Time Optimal Reset Control for Hard Disk Drive Servo Systems

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This paper proposes a design method of discrete-time optimal reset control with the reset time instants prespecified. With a base linear system designed conventionally, the discrete-time reset law design aims at improving the system transient responses. The design method can guarantee system stability, and the controller is obtained by solving Riccati equations. The proposed design method is applied to short-span and long-span track seeking of a hard disk drive servo system. Experimental results show that the proposed design is much more capable of improving transient response than traditional control design methods.

Index Terms—Discrete-time reset control systems, hard disk drive, linear quadratic regulation, reset law, servo systems.

I. INTRODUCTION

RESET control was first proposed by Clegg [1] to overcome limitations of linear control. This reset controller, termed Clegg integrator, consists of an integrator and a reset law that resets the output of the integrator to zero when its input vanishes. From the basic idea of reset control, one can see that reset control is helpful in reducing windup caused by integration. Moreover, a Clegg integrator has a similar magnitude-frequency response as a pure integrator but with \(51.9\)\% less phase lag. This favorable property helps to increase the phase margin of a system. In [2], Krishman and Horowitz developed a quantitative control design procedure of Clegg integrator. In [3], Horowitz and Rodenbaum generalized the concept of reset control to higher order systems. More details can be found in [4].

A lot of work has shown the advantages of reset control over linear control. For instance, in [5], an example is presented to show that reset control can achieve some control specifications that cannot be achieved by any ordinary linear control. Reference [6] shows that reset control can achieve much better sensor noise suppression without degrading disturbance rejection or losing margins. These advantages make reset control an important technique for performance improvement. See [7]–[9], for instance. Recently, reset control was introduced to hard disk drive (HDD) servo systems [10], [11].

There are in general two steps in reset control design [12]: linear compensator design and reset element design. Linear compensator is designed to meet all performance specifications other than output overshoot; then the reset element is designed to reduce the overshoot. As we know, a reset controller can improve closed-loop performance only when the reset law interacts well with the base linear system. In other words, if the reset controller is not appropriately designed, it may have little contribution to the performance improvement, or even cause performance degradation. For example, reset control may destroy the stability of the closed-loop system if it does not cooperate well with the base linear system.

The purpose of this paper is to propose a novel approach to reset control law design for a reset control system and apply the proposed method to improve the transient response of HDD servo systems. We focus on systems of which the base linear systems are already appropriately designed and the reset time instants are prespecified. The design of the reset law aims to minimize some performance index, and its solution is obtained by solving a Riccati equation. In the case of equidistant reset control, we show that the resulting reset law is time-invariant. This paper shows a discrete-time reset control law design method that is suitable to digital sampling control system and verified via experiment with an HDD servo system.

This paper is arranged as follows. In Section II, we set up the problem studied in this paper. In Section III, we propose the design method of the optimal reset control law. Section IV gives the application results of the proposed design method to HDD servo systems. Some concluding remarks are made in Section V.

II. PROBLEM SETTING

A typical reset control system is depicted in Fig. 1. The dynamics of the plant is described by

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx \tag{1}
\end{align*}
\]

where \(x \in \mathbb{R}^n\) is the state, \(u \in \mathbb{R}^m\) the control input, and \(y \in \mathbb{R}^p\) the output of the system. The zero-order-hold equivalent description of the plant (1) with sampling time \(T_s\) is given by

\[
\begin{align*}
x(i+1) &= A_dx(i) + B_du(i) \\
y(i) &= Cx(i) \tag{2}
\end{align*}
\]

where \(A_d = e^{AT_s}, B_d = \int_0^{T_s} e^{A(T_s-\tau)}d\tau\).

Define \(T_r\) as the reset period. We assume that the reset interval \(T_r\) is at every \(n\) sampling intervals, i.e., \(T_r = nT_s\), as illustrated below.

The notations \(k\) and \(i\) are used to represent the time instant \(t = kT_r + iT_s\) with \(i = 0, 1, 2, \ldots, n - 1\). We then introduce the following discrete reset controller

\[
\begin{align*}
z(k,i) &= Dz(k,i-1) + Ec(i-1) \\
z(k,0) &= p_k(x(k-1,n-1),z(k-1,n-1),r) \\
u(k,i) &= Gx(k,i) + Hz(k,i) + Me(k,i) \tag{3}
\end{align*}
\]
where $x \in \mathbb{R}^n$ is the dynamic state in the controller, $r$ is the reference signal, which is assumed to be constant, and $\rho_r(x, z, r)$ is the reset value at time instant $t = kT_r$ and is the so-called reset law. $D, E, G, H$, and $M$ are all constant matrices with compatible dimensions. $e(k, i) = r - y(k, i)$ is the tracking error.

Next we combine (2) and the reset controller (3) as follows:

$$
\begin{align*}
\vec{x}(k, i) &= \vec{A}\vec{x}(k, i-1) + \vec{B}r, \\
\vec{z}(k, 0) &= \rho_r(x(k-1, n-1), z(k-1, n-1), r), \\
\vec{y}(k, i) &= \vec{C}\vec{x}(k, i),
\end{align*}
$$

(4)

where $T(k, i) = (x(k, i)^T, z(k, i)^T)^T, \vec{C} = (C, 0_{p \times q})$

$$
\vec{A} = \begin{pmatrix}
A_d + B_d G - B_d M C & B_d M \\
-EC & D
\end{pmatrix},
$$

$$
\vec{B} = \begin{pmatrix}
B_d M \\
E
\end{pmatrix}.
$$

Before we introduce a cost function for the reset controller design, we need to make an assumption on the steady state of the system (4).

**Assumption 1:** For any $r \in \mathbb{R}^p$, there exists $\vec{x}_r = (x_r^T, z_r^T)^T \in \mathbb{R}^{n+q}$ such that $\vec{A}\vec{x}_r + \vec{B}r = \vec{x}_r, \vec{C}\vec{x}_r - r = 0$.

With the above assumption, the control input $u_r$ in the steady state is given by

$$
u_r = G\vec{x}_r + Hz_r.
$$

(5)

For each time instant $t = kT_r + iT_s$, we define a cost function $J(k, i)$ as

$$
J(k, i) = e^T(k, i)Q_{k,i}e(k, i) + (u(k, i) - u_r)^T R_{k,i} (u(k, i) - u_r).
$$

(6)

Here $Q_{k,i}$ and $R_{k,i}$ are positive semidefinite matrices. The optimal reset law (ORL) design problem considered in this paper is thus formulated as follows.

**Problem 1:** Design $\rho_r$, such that the resulting control system is asymptotically stable and meanwhile the cost function $J(\infty)$ is minimized, where

$$
J(\infty) = \sum_{k=0}^{\infty} \sum_{i=0}^{n-1} J(k, i).
$$

III. DISCRETE-TIME OPTIMAL RESET LAW DESIGN

In this section, we aim to solve the ORL problem stated in the previous section. It will be proved that this problem can be equivalently converted into a standard linear quadratic regulation (LQR) problem.

We make a coordinate transformation as follows:

$$
\begin{align*}
\xi(k, i) &= \vec{x}(k, i) - x_r, \\
\xi_z(k, i) &= z(k, i) - z_r,
\end{align*}
$$

(7)

where the steady-state values $x_r$ and $z_r$ are defined in Assumption 1. From (4), we have

$$
\begin{align*}
\xi(k, i) + 1 &= \vec{A}\xi(k, i), \\
\xi_z(k, 0) &= \vec{B}\xi(k, i), \\
\epsilon(k, i) &= -\vec{C}\xi_z(k, i),
\end{align*}
$$

(8)

where $\xi(k, i) = \begin{pmatrix} \xi_x(k, i) \\ \xi_z(k, i) \end{pmatrix}$ and $\vec{B} \xi(k, 0) = \rho(k, 0) - z_r$. From (3) and (5), it follows that

$$
u(k) - u_r = [G - MC \ H]^T \begin{pmatrix} \xi_x(k) \\ \xi_z(k) \end{pmatrix}.
$$

(9)

Let $N = [G - MC \ H]$. For a reset interval $(iT_r, (i+1)T_r]$, the cost function should be

$$
\sum_{i=0}^{n-1} J(k, i) = \sum_{i=0}^{n-1} e^T(k, i)Q_{k,i}e(k, i) + (u(k, i) - u_r)^T R_{k,i} (u(k, i) - u_r) =
$$

$$
\sum_{i=0}^{n-1} \xi_x^T(k, i)C^T Q_{k,i} \xi_z(k, i) + \xi_z^T(k, i)N^T R_{k,i} N \xi_z(k, i) =
$$

$$
\sum_{i=0}^{n-1} \xi_x^T(k, i)(\vec{C}^T Q_{k,i} \vec{C} + N^T R_{k,i} N) \xi_z(k, i) =
$$

$$
\xi_z^T(k, 0) \sum_{i=0}^{n-1} (\vec{A}^T)^i(\vec{C}^T Q_{k,i} \vec{C} + N^T R_{k,i} N) (\vec{A}^T)^i \xi_z(k, 0) =
$$

$$
\xi_z^T(k, 0) \Theta_k \xi_z(k, 0),
$$

(10)

with $\Theta_k = \sum_{i=0}^{n-1} [(\vec{A}^T)^i(\vec{C}^T Q_{k,i} \vec{C} + N^T R_{k,i} N) (\vec{A}^T)^i]$. Hence $J(\infty)$ is the sum of $\xi_z^T(k, 0) \Theta_k \xi_z(k, 0)$ at each reset time instant $kT_r$ and given by

$$
J(\infty) = \sum_{k=0}^{\infty} \xi_z^T(k, 0) \Theta_k \xi_z(k, 0),
$$

(11)

On the other hand, with the reset interval $T_r$ as sampling interval, the system (8) can be written as

$$
\begin{pmatrix}
\xi_x(k+1, 0) \\ \xi_z(k+1, 0)
\end{pmatrix} = (\vec{A})^n \begin{pmatrix}
\xi_x(k, 0) \\ \rho(k, 0)
\end{pmatrix}.
$$

(12)

Partition $(\vec{A})^n$ as follows:

$$
(\vec{A})^n = \begin{pmatrix}
\Gamma_A & 0 \\
0 & \Gamma_B
\end{pmatrix}.
$$

(13)
Then we have

\[ \xi_e(k+1,0) = \Gamma_A \xi_e(k,0) + \Gamma_B \tilde{p}(k,0), \quad (14) \]

Note that (14) is simply a linear discrete system with state variable \( \xi_e \) and control input \( \tilde{p} \). Consider the following ORL problems of (14).

**Problem 2:** Design a control sequence \( \rho(i), i = 0, 1, \ldots \), for system (14) such that the resulting system is asymptotically stable and the following quadratic performance index is minimized:

\[ J(\infty) = \sum_{k=0}^{\infty} \begin{pmatrix} \xi_e(k,0) \\ \tilde{p}(k,0) \end{pmatrix}^T \Theta_k \begin{pmatrix} \xi_e(k,0) \\ \tilde{p}(k,0) \end{pmatrix}. \]

**Proposition 1:** Suppose that there exist positive numbers \( \varepsilon > 0 \) such that \( \lambda_{\text{min}}(\Gamma_B^2 \Gamma_B) \geq \varepsilon > 0 \). Then ORL Problem 1 is equivalent to LQR Problem 2.

**Proof:** We only need to prove that under the above condition, system (14) is asymptotically stable if and only if system (8) is asymptotically stable. It is obvious that if system (8) is asymptotically stable, then system (14) is asymptotically stable.

In the following, we assume that system (14) is asymptotically stable. Then we have

\[ \lim_{k \to \infty} \xi_e(k,0) = 0, \]
\[ \lim_{k \to \infty} \Gamma_B \tilde{p}(k,0) = 0. \]

According to condition \( \lambda_{\text{min}}(\Gamma_B^2 \Gamma_B) \geq \varepsilon > 0 \), we have

\[ \lim_{k \to \infty} \tilde{p}(k,0) = 0. \]

Note that \( \xi_e(k,0) = \tilde{p}(k,0) \); we have

\[ \lim_{k \to \infty} \xi_e(k,0) = 0 \]

and thus system (8) is asymptotically stable.

The optimal reset law that stabilizes system (1) and minimizes \( J(\infty) \) is given by

\[ \rho^*(k,0) = -K(x(k,0) - x_r) + z_r, \quad (15) \]

where \( K \) is determined by

\[ K = (\Gamma_B^2 S \Gamma_B + R)^{-1} (\Gamma_B^2 S \Gamma_A + T). \]

\[ S \] is the solution of the Riccati equation

\[ S = \tilde{Q} + \Gamma_A^T S \Gamma_A - \Gamma_A^T S \Gamma_B (\Gamma_B^2 S \Gamma_B + R)^{-1} \Gamma_B^2 S \Gamma_A. \]

Furthermore, the minimum of \( J(\infty) \) is given by

\[ J^*(\infty) = (x(0) - x_r)^T S(x(0) - x_r). \]

**IV. APPLICATION TO AN HDD SERVO SYSTEM**

**A. System Modeling**

The frequency responses of an HDD voice coil motor (VCM) actuator have been measured using a laser Doppler vibrometer and a dynamic signal analyzer. Based on the measured frequency responses of the actuator, its transfer function is obtained as follows:

\[
P(s) = \frac{2.4819 \times 10^6}{s^2 + 452.4s + 5.685 \times 10^8} \times \frac{s^2 + 1100s + 1922 \times 10^8}{s^2 + 1936s + 2.603 \times 10^8} \times \frac{s^2 + 14830s + 8.795 \times 10^8}{s^2 + 5931s + 2.188 \times 10^9}
\]

(19)

whose frequency responses have been plotted in Fig. 2.

We apply notch filters to compensate the resonance modes. The system is then simplified as a second-order system. The model of the nominal plant is described by

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -ax_1 - bx_2 + cu \\
y &= x_1
\end{align*}
\]

(20)

where \( a = 5.6849 \times 10^3, b = 407.1, c = 7.3647 \times 10^7 \). Our target is to design a seeking controller for the plant to have fast positioning without overshoot.

**B. Reset Control Law Design**

The reset controller is given by

\[
\begin{align*}
z(k, i) &= Dz(k, i - 1) + Ez(i - 1) \\
z(k, 0) &= \rho_n(z(k - 1, n - 1), z(k - 1, n - 1), r) \\
a(k, i) &= \alpha z(k, i) + \beta x_2(k, i) + \gamma z(k, i)
\end{align*}
\]

(21)

with \( D = 1, E = T_s, \alpha = 0.2, \beta = 1 \times 10^{-4} \), and \( \gamma = 8000 \).

In the optimal reset law design, first we need to design the baseline linear controller given by the first and the third equations in (21). It is proportional integral derivative (PID) controller, and the design target is to achieve swift movement de-
manding and thus shorter rise time regardless of its large overshoot in the displacement. Then we use reset law to suppress the overshoot, while in the rising period, the performance is similar to that by the PID controller. Thus the final optimal reset control will achieve both short rise time and little overshoot, which is, however, difficult to achieve with a PID controller alone.

For the reset law design, we assume \( R_{k_s} \) and \( Q_{k_s} \) are time-invariant. The reset law is given by

\[
\rho_k^* = -k_1(x_1 - r) - k_2 x_2 + \frac{a}{c_f} r, \tag{22}
\]

When the reset interval \( T_r = 2 \times T_s \), by solving Riccati equation (16) and (17), we have \( k_1 = -6.1967 \times 10^{-3} \) and \( k_2 = -3.2968 \times 10^{-5} \). In (22), the velocity \( x_2 \) is replaced with its estimated value \( \hat{x}_2 \). A state observer is needed to obtain the estimated velocity.

C. Experimental Results

The discrete optimal reset control law is verified via implementation in the HDD servo system, and the seeking performance is investigated. Fig. 3 shows the seeking result for \( r = 0.25 \mu m \) with comparison to the baseline linear PID control. It is seen that the rising times of the linear controller and the reset controller are similar, but when reaching the target, the response with the ORL method is swiftly settled to the target with less overshoot. The settling time is 0.33 ms.

Fig. 4 shows the seeking performance for \( r = 4 \mu m \). The performance of this longer span seeking with the reset control is also much better than that with the baseline linear PID control. No overshoot is noticed, and the settling time is less than 0.5 ms. The control effort of the ORL method has much more efficiency since it is minimized via the cost function, including the control input. Then we proved that the ORL design problem is equivalent to the LQR problem. Based on this, the optimal reset laws were given in terms of Riccati equations. The proposed design method of the reset control law is very simple, and the resultant reset law can be easily implemented online. The optimal reset control has been applied to the seeking control of hard disk drive servo systems. Both short-span and long-span seeking performances were evaluated. Experimental results show that the proposed method can achieve better seeking performance.

V. CONCLUSION

In this paper, we have studied discrete optimal reset law design of reset control systems. First, we transferred closed-loop reset control system to discrete linear system with reset value as control input. Then we proved that the ORL design problem is equivalent to the LQR problem. Based on this, the optimal reset laws were given in terms of Riccati equations. The proposed design method of the reset control law is very simple, and the resultant reset law can be easily implemented online. The optimal reset control has been applied to the seeking control of hard disk drive servo systems. Both short-span and long-span seeking performances were evaluated. Experimental results show that the proposed method can achieve better seeking performance.

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