Outage Performance of Underlay Multi-hop Cognitive Relay Networks with Energy Harvesting

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Abstract—This letter studies the outage performance of multi-hop cognitive relay networks with energy harvesting in underlay paradigms, wherein the secondary users are powered by a dedicated power beacon (PB) and their transmit powers are subject to the harvested energy from PB and the interference constraint from the primary user. We derive the exact outage probability for Rayleigh block fading and prove that the outage probability is monotonically decreasing with respect to the transmit power of PB. Furthermore, we derive the asymptotic outage probability to study the outage saturation phenomenon and propose an iterative algorithm that jointly optimizes the transmit power of PB and the harvest-to-transmit ratio to approximate the minimum outage probability. Simulation results validate the theoretical results.

Index Terms—multi-hop, underlay, cognitive relay networks, energy harvesting, outage probability.

I. INTRODUCTION

A.

S energy harvesting can virtually provide perpetual energy supply for energy-constraint networks without manual battery recharging or replacement, energy harvesting cognitive relay networks (EH-CRNs) have recently attracted a great deal of attention. In interweave paradigms, the secondary users (SU) first harvest energy from ambient energy sources [1] or the RF signals of active primary users (PUs) [2], [3] and then opportunistically access the licensed channels when PUs are detected as inactive. In overlay paradigms, SUs harvest energy from PUs and transmit data for both PUs and SUs provided that there are excellent cooperations between PUs and SUs [4]. In underlay paradigms, SUs harvest energy from PUs [5], [6] or other SUs [7], and transmit as long as the interference to PUs is no longer than a prescribed threshold. More recently, overlay and underlay cognitive wireless powered networks are analyzed and compared in [8]. Moreover, a hybrid interweave and underlay cognitive radio is studied in [9].

Different from the above works, this letter introduces the power beacon (PB) [10] to power SUs for the multi-hop transmission in underlay EH-CRNs, since PB can efficiently power SUs with low deployment cost while achieving decoupled simultaneous wireless information and power transfer (SWIPT) without any backhaul links [11], [12]. Specifically, SUs first harvest energy from the RF signals of a dedicated PB on the common control channel, and then transmit concurrently with

Fig. 1. System model.

PU over the same licensed channel using the harvested energy. The transmit powers of SUs are set by jointly considering the harvested energy from PB and the interference constraint from PU, which can maximize the transmit powers under the condition that PU is sufficiently protected. We derive the closed-form outage probability for the underlay multi-hop EH-CRNs and provide insights into the effect of energy conversion efficiency, harvest-to-transmit ratio (HTR), number of hops, transmit power of PB and interference constraint from PU on the outage performance.

The main contributions of this letter can be summarized as follows: (i) we derive exact and asymptotic outage probabilities for multi-hop EH-CRNs over Rayleigh block fading; (ii) we prove that the outage probability is monotonically decreasing with respect to PB’s transmit power and further investigate the outage saturation phenomenon; (iii) we propose an iterative algorithm to obtain a near-optimal outage probability by jointly optimizing PB’s transmit power and HTR.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a multi-hop EH-CRN, whose typical application is sensor network, with $K+1$ SUs (e.g., sensors) in an underlay paradigm. The PU transmitter is far away from SUs and does not impose any interference on SUs [7], while the PU receiver must be sufficiently protected by controlling the transmit powers of SUs. Each SU is equipped with an energy harvester and operates in half-duplex mode with which SU can only transmit, receive or harvest. All SUs are powered by a dedicated PB and perform multi-hop transmission to lower each hop’s transmit power. With the time division multiple access mechanism, the source $SU_1$ transmits data to the destination $SU_{K+1}$ serially via immediate decode-and-forward relays. By time switching [13], the whole communication within a block of time $T$ is divided into two phases, namely, energy harvesting and data transmission.

In the energy harvesting phase with duration $\tau$ ($0 < \tau < T$), all SUs harvest energy from the RF signals of PB on the common control channel, wherein the noise energy is negligible compared to the transmit power of PB $P_t$. At the end of this phase, the harvested energy stored in the capacitor of $SU_k$ ($k = 1, \ldots, K+1$) is $\xi_k P_t d_{Ek}^{\alpha}$, where $0 \leq \xi_k \leq 1$ is the energy conversion efficiency depending on the design of
energy harvester, $\alpha$ is the path loss exponent, $d_{E_k}$ and $g_{E_k}$ are the distance and the channel power gain between PB and SU$_k$.

The data transmission phase with duration $T - \tau$ is equally divided into $K$ time slots for the $K$-hop transmission on the licensed channels. Note that the energy harvesting from PB and the data transmission among SUs are sufficiently separated in different channels since $P_t$ should be large enough to power SUs with sufficient bandwidth according to [10]. Then, subject to the harvested energy from PB and the interference constraint from PU, the transmit power of SU$_k$ is set as

$$P_k = \min \left( \frac{K_{E_k} P d_{E_k}^\alpha g_{E_k}}{T - \tau}, \frac{I_p}{d_{ik}^\alpha g_{ik}} \right),$$  

where $I_p$ is the peak interference power that PU can tolerate, $d_{ik}$ and $g_{ik}$ are the distance and the channel power gain between SU$_k$ and PU, respectively. Furthermore, we denote $\eta = T - \tau$, as HTR and assume $\xi_k = \ldots = \xi_{k+1} = \xi$ without loss of generality. Note that (1) ignores the circuit energy loss of generality. Note that (1) ignores the circuit energy consumption [11], and the remaining energy before the current block due to the leakage of capacitor [5].

As a result, the signal-to-noise ratio (SNR) for the $k$-th hop transmission is calculated as

$$\gamma_k = \frac{P_k d_{ik}^\alpha g_{ik}}{N_0},$$  

where $N_0$ is the noise power, $d_{ik}$ and $g_{ik}$ are the distance and the channel power gain between SU$_k$ and SU$_{k+1}$, respectively.

In this paper, all the channels suffer from independent non-identically distributed (i.i.d) Rayleigh block fading with variance $1/\lambda_j$ for $i = E, I, D; j = k$. Thus, the channel power gain $g_{ij}$ is an exponentially distributed random variable with probability density function (PDF) $f_{g_{ij}}(t) = \lambda_{ij} \exp(-\lambda_{ij}t)$ and cumulative distribution function (CDF) $F_{g_{ij}}(t) = 1 - \exp(-\lambda_{ij}t)$. The required channel state information can be perfectly obtained by PB and SUs through channel training and estimation, pilot sensing, direct feedbacks from PU and SUs, or even indirect feedbacks from a band manager [6].

### III. OUTAGE PERFORMANCE ANALYSIS

#### A. Exact Analysis

The outage probability of a multi-hop EH-CRNs is defined as the probability that the end-to-end capacity is less than a rate threshold $R$ and can be mathematically expressed as

$$P_{out} = P\left( \frac{1}{K(1+\eta)} \log_2(1 + \min_{k=1,\ldots,K} \gamma_k) \leq R \right)$$

$$= 1 - \prod_{k=1}^K (1 - P(\gamma_k \leq \gamma_{th})), \quad (3)$$

where $1/(1+\eta)$ is set due to the fact that $F^{-1}$ is divided by $K$ hops, and $\gamma_{th} = 2^{KR(1+\eta)} - 1$ is the SNR threshold.

To calculate $P_{out}$, we need to derive the CDF of $\gamma_k$, i.e., $F_{\gamma_k}(\gamma_{th})$, which is given by the following proposition.

**Proposition 1:** The closed-form CDF of $\gamma_k$ is given by

$$F_{\gamma_k}(\gamma_{th}) = 1 - \sqrt{\Delta_{1k}K_1(\sqrt{\Delta_{1k}})} + \sum_{k=1}^{\Delta_{1k}} K_1(\sqrt{\Delta_{1k}}),$$  

where $\Delta_{1k} = \Delta_{1k} + \Delta_{2k}$, $\Delta_{1k} = \frac{4\lambda_{E_k}\lambda_{E_k}d_{E_k}^\alpha g_{E_k}}{K_0^2F_k d_{ik}^\alpha g_{ik}}$, $\Delta_{2k} = \frac{4\lambda_{E_k}\lambda_{E_k}P_t d_{E_k}^\alpha g_{E_k}}{K_0^2F_k d_{ik}^\alpha g_{ik}}$, and $K_1(t)$ is the modified Bessel function of the second kind with order $v$.

**Proof:** Please refer to Appendix for details.

#### B. Asymptotic Analysis

As the harvested energy is closely related to the transmit power of PB, we give out the following proposition to definitely show the positive impact of PB on the outage performance and gain insight into the outage saturation phenomenon.

**Proposition 2:** Given $\xi$ and $\eta$, $P_{out}$ for $K$-hop EH-CRNs with the prescribed $I_p$ is monotonically decreasing with respect to $P_t$ until an outage floor, which is the asymptotic outage probability when $P_t \to \infty$, i.e.,

$$P_{out} \approx 1 - \prod_{k=1}^K \left( \frac{\Delta_{2k}}{\Delta_{1k}} \right).$$  

**Proof:** By utilizing [14, 8.486.12] for the derivative of $K_{\xi}(t)$, we can derive the partial derivative of $P_{out}$ with respect to $P_t$ as

$$\frac{\partial P_{out}}{\partial P_t} = \sum_{i=1}^K \prod_{k=i}^K (1 - F_{\gamma_k}) \frac{\partial F_{\gamma_k}}{\partial P_t},$$  

with

$$\frac{\partial F_{\gamma_k}}{\partial P_t} = \frac{\Delta_{1k}}{2P_t} \left( K_0(\sqrt{\Delta_{1k}}) - K_0(\sqrt{\Delta_{1k}}) \right).$$

It is observed that $\frac{\partial F_{\gamma_k}(\gamma_{th})}{\partial P_t} < 0$, since $K_{\xi}(t)$ is a monotonically decreasing function of $t$. Thus, $\frac{\partial P_{out}}{\partial P_t} < 0$, which indicates that $P_{out}$ is monotonically decreasing in $P_t$. When $P_t \to \infty$, both $\Delta_{1k}$ and $\Delta_{2k}$ approach 0. According to [15, 9.6.9], when $v$ is fixed, $K_v(t) \approx \frac{1}{2\Gamma(v)(\frac{t}{2})^{-v}}$, where $\Gamma(v)$ is the Gamma function. Thus, the approximation of (4) can be derived as

$$F_{\gamma_k}(\gamma_{th}) \approx \frac{\Delta_{1k}}{\Delta_{1k}},$$

which further results in (5) by substituting it into (3) with some algebraic operations.

Similarly, we can prove that $P_{out}$ is also monotonically decreasing with respect to $\xi$. Proposition 2 shows that the outage performance can be enhanced by increasing $P_t$. When $P_t \to \infty$, the asymptotic outage probability is independent of the harvested energy. The reason is given as follows: when $P_t \to \infty$, $P_t = \min(K, \xi, \eta, (\frac{K}{d_{E_k}^\alpha g_{E_k}} + \frac{I_p}{d_{ik}^\alpha g_{ik}}) \approx I_p/d_{ik}^\alpha g_{ik}$ with probability 1, which indicates that the interference constraint becomes the dominating factor to determine the transmit power of SU$_k$. In other words, the outage probability will saturate when the harvested energy is sufficiently large. The outage saturation phenomenon indicates that overmuch energy harvesting or wireless energy transfer cannot improve the outage performance significantly when the interference constraint is prescribed in practice. As a result, the asymptotic outage probability can be regarded as an outage floor which will guide us to set the transmit power of PB properly.

#### C. Optimization

As $P_{out}$ is significantly affected by the network setup ($P_t, \eta$), we formulate the following optimization problem.

$$\min_{P_t, \eta} \quad P_{out} = 1 - \prod_{k=1}^K (1 - F_{\gamma_k}(\gamma_{th})), \quad (8)$$

s.t. $P_t \geq 0, \quad \eta > 0$.

Then, instead of directly solving the two-variable problem, we decouple it into two single-variable problems and iteratively solve them as Algorithm 1.
1) Power Optimization: By Proposition 2, the minimum outage probability is achieved only when $P_t$ is set infinitely large, which is impracticable due to hardware limitations. However, we can employ an efficient way to find a suboptimal transmit power of PB $P_t^*$ that is practicable and can achieve a near-optimal $P_{out}$ for given $\eta$. We can calculate the second-order partial derivative with respect to $P_t$ as

$$\frac{\partial^2 P_{out}}{\partial P_t^2} = K \sum_{k=1}^K \left( \prod_{i \neq j} (1 - F_{\gamma_i} \frac{\partial^2 F_{\gamma_j}}{\partial P_t^2} - \sum_{i \neq j} K \frac{\partial F_{\gamma_i}}{\partial P_t} \frac{\partial F_{\gamma_j}}{\partial P_t^2} \right)$$

(9)

with $\frac{\partial^2 F_{\gamma_i}}{\partial P_t^2} = \frac{\partial}{\partial P_t} \left( G(\Delta_k) - G(\Delta_0k) \right)$, and $G(\tau) = 4K_0(\sqrt{\tau} - \sqrt{\Delta k_1(\sqrt{\tau})}$ which is verified to be a first decreasing and then increasing function with respect to $\tau$.

Then, we can conclude that $\frac{\partial^2 F_{\gamma_i}}{\partial P_t^2} > 0$ and $\frac{\partial^2 F_{\gamma_i}}{\partial P_t^2} \left( \frac{\partial F_{\gamma_i}}{\partial P_t} < 0 \right)$ is monotonically increasing when $P_t$ is beyond the point that satisfies $G(\Delta_k) = G(\Delta_0k)$. Thus, there must be an inlexion point $P_t^*$ fulfilling $\frac{\partial^2 P_{out}}{\partial P_t^2} = 0$, at which the curve of $P_{out}$ changes from concave to convex. Then, we employ Newton’s method to find $P_t^*$. The initial point $P_t^0$ can be selected from the neighborhood of $P_t$, i.e., $P_t^0 \in U(P_t, \delta)$, where $U(P_t, \delta) \equiv \{ t \mid |t - P_t| < \delta, t \neq P_t \}$. The stopping criterion is set as $|\frac{\partial P_{out}}{\partial P_t}| \leq \epsilon$. Both $\delta$ and $\epsilon$ are sufficiently small positive constants.

2) HTR Optimization: Given $P_t$, we solve the equivalent problem min $P_{out}$ instead of directly optimizing $\eta$ since $\eta = \frac{P_t}{\tau}$. In contrast to $P_t$, $P_{out}$ is not a monotonic function of $\tau$.

We can differentiate $P_{out}$ with respect to $\tau$ as

$$\frac{\partial P_{out}}{\partial \tau} = \sum_{k=1}^K \prod_{i \neq j} (1 - F_{\gamma_i}(\eta_k)) \left( \Omega_{1k} \frac{\partial \Delta_{\gamma_k}}{\partial \tau} + \Omega_{2k} \frac{\partial \Delta_{\gamma_k}}{\partial \tau} \right),$$

(10)

with $\Omega_{1k} = \frac{1}{2}K_0(\sqrt{\Delta_k}) - \frac{1}{\Delta_k^{2/3}}K_0(\sqrt{\Delta_0k}) + \frac{1}{\Delta_k^{1/3}}K_1(\sqrt{\Delta_0k})$ and $\Omega_{2k} = -\frac{1}{\Delta_k^{2/3}}K_0(\sqrt{\Delta_0k}) - \frac{1}{\Delta_k^{1/3}}K_1(\sqrt{\Delta_0k})$. It is observed that $\Omega_{1k} \leq 0 \leq \Omega_{2k}$ since $K_k(t) > 0$ is monotonically decreasing in $t$. Moreover, it can be verified that $\frac{\partial \Delta_{\gamma_k}}{\partial \tau} < 0$ and there is a unique $\tau$ smaller than which $\frac{\partial \Delta_{\gamma_k}}{\partial \tau} < 0$. Thus, we reasonably deduce that there is an optimal energy harvesting duration $\tau^*$ that satisfies $\frac{\partial P_{out}}{\partial \tau} = 0$ and yields the minimum outage probability. Fig. 3 will validate our deduction. However, to the best of our knowledge, there is no closed-form expression for $\tau^*$ due to the complexity of $P_{out}$. Instead, we can evaluate $\tau^*$ numerically by a one-dimensional search with the help of Matlab. Then, the optimal HTR is $\eta^* = \frac{\tau^*}{P_t}$.

IV. SIMULATION RESULTS

In this section, we validate the theoretical results by Monte Carlo simulations with 10^5 runs. Without loss of generality, the source and the destination are placed at $[-2, 0]$ and $[2, 0]$ with the relays equally scattered between them on x-axis, while PB and PU are placed at $[0, 2]$ and $[0, -2]$ on y-axis. The channel parameters are set as $\alpha = 2$ and $\lambda_{E^{max}} = \lambda_{D^{max}} = \lambda_{D^{min}} = 1$ for $k = 1, \ldots, K + 1$. The noise power is set as $N_0 = 1$, while both $P_t$ and $I_p$ are normalized by $N_0$. In addition, $\xi = 0.8$, $T = 100$ ms, $R = 0.5$ bit/s/Hz, $\delta = 10^{-1}$, and $\epsilon = 10^{-3}$.

Fig. 2 investigates the impact of $P_t$ on $P_{out}$ for a three-hop EH-CRN. As shown, there is a good agreement between the theoretical and the simulation results. When $P_t$ is small, $P_{out}$ approaches 1 as the harvested energy is negligible. When we elevate $P_t$ or $\xi$, $P_{out}$ decreases monotonically. The reason is that the harvested energy grows with the increase of $R_t$ or $\xi$, and therefore the transmit powers of SUs are enhanced provided that the interference at PU is no larger than $I_p$. When $P_t$ becomes large enough, $P_{out}$ with the same $I_p$ converges to the same outage floor, i.e., the asymptotic outage probability, which validates Proposition 2. It is also obvious that the curve of $P_{out}$ is first concave and then convex in $P_t$, which validates the conclusion in Section III. C. Furthermore, the outage floor increases when the interference constraint becomes strict, i.e., $I_p$ decreases. In addition, for the same $\xi$, $P_{out}$ with interference constraint is always higher than that without constraint.

Fig. 3 clearly shows the relationship between $P_{out}$ and $\tau$. For given $K$, $P_{out}$ first decreases to a minimum when $\tau$ attains the optimum and then increases along with $\tau$ until $P_{out}$ approaches 1, which indicates that $P_{out}$ is not monotonic in $\tau$. In addition, there are intersections between the curves of different $K$’s, which indicates that $P_{out}$ is not monotonic in $K$ for given $\tau$. We can observe that when $\tau$ is small (e.g., $\tau < 40$ ms), namely long duration is left for transmission, multi-hop is superior to single-hop, and vice versa when $\tau$ is large (e.g., $\tau > 60$ ms). For given $\tau$, the optimal $K$ minimizing $P_{out}$ can be obtained by a one-dimensional search as $K$ is a finite integer.

Algorithm 1 Find $(P_t^*, \eta^*)$ to approximate the optimal $P_{out}$

1: Input: $K$, $T$, $\xi$, $\delta$, $\epsilon$, and $I_p$.
2: Initialization: $n \leftarrow 0$, $\forall \tau(n) \in (0, T)$.
3: repeat: Given $\tau(n)$, find $P_t^*$ by solving $\frac{\partial P_{out}}{\partial P_t} = 0$ and select $\forall P_t(n) \in (U(P_t, \delta), m = 0, P_m = P_t^{(0)}$.
4: repeat: $P_{out}^{(m+1)} = P_{out}^{(m)} - \frac{\partial P_{out}}{\partial P_t} \bigg|_{P_t=P_t^{(m)}}$, $m \leftarrow m + 1$
5: until $|\frac{\partial P_{out}}{\partial P_t} |_{P_t=P_t^{(m)}} | \leq \epsilon$.
6: $P_t^* = P_t^{(m)}$, $P_{out}^{(n+1)} = P_t^*$.
7: Given $P_{out}^{(n)}$, find $\tau^*$ fulfilling $\frac{\partial P_{out}}{\partial \tau} \bigg|_{\tau=\tau^*} = 0$ by a one-dimensional search.
8: $\tau^{(n+1)} = \tau^*$, $n \leftarrow n + 1$.
9: until $P_{out}^{(n)}$, $P_{out}^{(n)} \geq P_{out}^{(n-1)}, \tau^{(n-1)}$.
10: Output: $P_t^* = P_t^{(n)}$, $\eta^* = \frac{\tau^*}{P_t^*}$.
In this letter, we derived exact and asymptotic closed-form outage probabilities for underlay multi-hop EH-CRNs over i.n.i.d Rayleigh block fading, where SUs are powered by a dedicated PB and their transmit powers are determined by the harvested energy and the interference constraint. Simulations results verified that the outage probability is monotonically decreasing to an outage floor with the increase of PB’s transmit power and a near-optimal outage probability can be achieved by jointly optimizing PB’s transmit power and HTR.

V. CONCLUSION

Then, the unconditional CDF marginalized out $Y$ is calculated as

$$F_{\alpha}^{(2)} = \sqrt{\Delta_{2\kappa} K_1(\sqrt{\Delta_{2\kappa}})} - \frac{\Delta_{2\kappa}}{\Delta_{2\kappa}} K_1(\sqrt{\Delta_{2\kappa}}), \quad (15)$$

where (14, eq.(3.324.1)) is again utilized for the integral during the derivation.

Finally, by substituting (13) and (15) into (11), we obtain (4) and complete the proof.

REFERENCES


